Latency exploitation in Wavelet Based Multirate Circuit Simulation

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Summary. We present a further improvement of the wavelet multirate circuit simulation. In the new algorithm we use different grids for the approximation of the solution on different circuit parts. In particular, for circuits with latencies the grids can be much sparser, which results in the reduction of the overall problem size and leads to a faster simulation.

1 The multi-rate circuit simulation problem

We consider circuit equations in the charge/flux oriented modified nodal analysis (MNA) formulation, which yields a mathematical model in the form of a system of differential-algebraic equations (DAEs):

$$\tfrac{d}{dt} q\big(x(t)\big) + g\big(x(t)\big) = s(t). \tag{1}$$

To separate different time scales the problem is reformulated as a multi-rate PDAE, i.e.,

$$\Big(\tfrac{\partial}{\partial \tau} + \omega(\tau)\,\tfrac{\partial}{\partial t}\Big) q\big(\hat{x}(\tau,t)\big) + g\big(\hat{x}(\tau,t)\big) = \hat{s}\big(\tau,t\big) \tag{2}$$

with mixed initial-boundary conditions $x(0,t) = X_0(t)$ and $x(\tau,t) = x(\tau,t+P)$. A solution of the original circuit equations can be found along certain characteristic lines [3]

Discretization with respect to τ (Rothe method) using a linear multi step method results in a periodic boundary value problem in t of the form

$$\begin{aligned} \omega_k \tfrac{d}{dt} q_k\big(X_k(t)\big) + f_k(X_k,t) &= 0, \\ X_k(t) &= X_k(t+P), \end{aligned} \tag{3}$$

where $X_k(t)$ is the approximation of $\hat{x}(\tau_k,t)$ for the k-th time step τ_k (cf. [1,3]). The periodic boundary value problem (3) can be solved by several methods, as Shooting, Finite Differences, Harmonic Balance, etc. Here, we consider the spline wavelet based method introduced by the authors in [1]. One problem of traditional methods is that all signals in the circuit are discretized over the same grid. This can pose a problem if different signal shapes are present in the circuit, which may be approximated more efficient if individual grids are used for each of the signals. As an example we consider a chain of 5 frequency dividers (as part of a PLL). In each step the frequency is reduced by a factor 2 as one can see in Fig. 1. Obviously, for the low frequency signals towards the end of the divider chain a much sparser grid would be sufficient for an accurate representation, in comparison to the high frequency input signal.

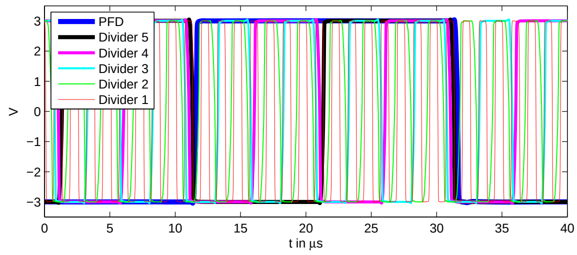

Fig. 1. Several signals in a frequency divider chain as part of a PLL.

2 Division into subcircuits and connections

Although the representation of each signal over its own individual grid seems to give maximal flexibility, this approach leads to several problems, which makes the simulation inefficient. Therefore, we consider groups of signals with similar shape appearing in a part of the circuit. The circuit is divided into N subcircuits which are connected at common nodes. Two facilitate different expansions of signals on the subcircuits we replace each common node by a pair of nodes connected by a perfect conductor. Namely, we introduce the "connection" $C^{k,\ell}_{\mu,\nu}$, if the μ-th node of subcircuit is identified with the ν-th node in subcircuit ℓ, as one can see in Fig. 2. Thus, we have the current through the connection $C^{k,\ell}_{\mu,\nu}$, as additional unknowns i^k_μ and i^ℓ_ν for each of the two involved subcircuits, as well as the equations

$$u^k_\mu(t) - u^\ell_\nu(t) = 0 \quad \text{and} \quad i^{k(t)}_\mu - i^\ell_\nu(t) = 0. \tag{4}$$

in addition to the circuit equations

$$\tfrac{d}{dt} q^k\big(x^k(t)\big) + g^k\big(x^k(t),t\big) = 0, \qquad k = 1,\dots,N \tag{5}$$

of the N subcircuits.

Using the Rothe's method on the multirate PDAE's will yield a system of DAE's as introduced in (5) and (4). Therefore we consider the solution of the periodic problem for the above DAE's.

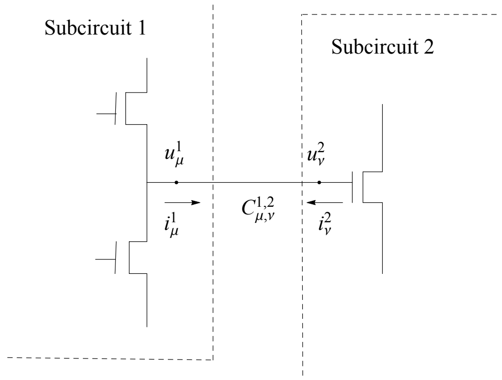

Fig. 2. Splitting of a circuit into subcircuits witch connections.

3 Spline Galerkin discretization and wavelet based adaptivity

Our goal is to approximate the solution of the equations (4) and (5) by spline functions as it was done in [1]. However, we use a different spline representations $x^i(t) = \sum_{k=1}^{n_i} c_k^i \varphi_k^i(t)$, $i = 1, \ldots, N$, for each of the subcircuits. A Petrow-Galerkin discretization yields the nonlinear system of equations

$$\int_{\tau_{\ell-1}^i}^{\tau_\ell^i} \tfrac{d}{dt} q^i\big(x^i(t)\big) + g^i\big(x^i(t), t\big)\, dt = 0, \qquad \ell = 1, \ldots, n_i,$$

for each subcircuit and

$$\int_{\tau_{\ell-1}^i}^{\tau_\ell^i} u_\mu^i(t) - u_\nu^j(t)\, dt = 0, \qquad \ell = 1, \ldots, n_i \qquad (6)$$

$$\int_{\tau_{\ell-1}^j}^{\tau_\ell^j} i_\mu^i(t) + i_\nu^j(t)\, dt = 0, \qquad \ell = 1, \ldots, n_j \qquad (7)$$

for each connection $C_{\mu,\nu}^{i,j}$ between subcircuits. The splitting points τ_ℓ^i are chosen in close relation to the spline grid.

The wavelet based coarsening and refinement methods described in [1, 2] are used to generate adaptive grids for an efficient signal representation.

4 Numerical test

Fig. 3 shows the spline grid generated for the classical algorithm using the same grid for all signals. We have plotted the grid points t_i against their index i, which allows to recognize the local density of the grid.

The grids used in our new multiple grid method can be seen in Fig. 4. Obviously, one gets much better adapted, smaller grids for the lower frequency signals. This leads to a reduction of the total number of equations from roughly 130,000 to 85,000. The number of nonzeros in the Jacobian for Newton's method is reduces from 5,000,000 to 2,500,000. Consequently the

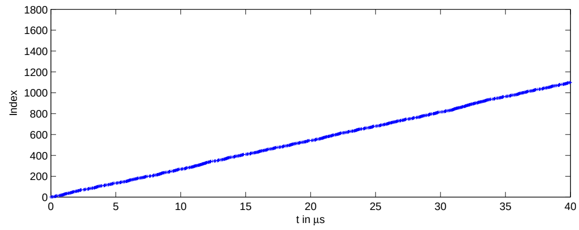

Fig. 3. Grid of the single grid method.

time for assembling resp. solving the linear system was reduced from 4s to 2s resp. 8s to 4s.

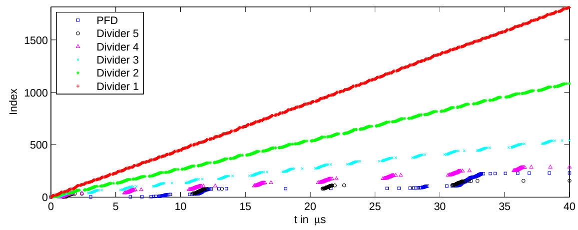

Fig. 4. Several signals in a frequency divider chain as part of a PLL.

A further effect is that the larger the nonlinear system is harder to solve by Newton's method, which results in more Newton iterations as well as shorter envelope time steps. Thus, an envelope simulation with a frequency modulated signal over 0.3s worked well for the multiple grid method and was done in 37min. A similar simulation by the single grid method needed almost 5 hours.

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References

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