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Reduced Basis Methods for Time-Harmonic Maxwell's Equations with Stochastic Coefficients

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Abstract

The Reduced Basis Method generates low-order models of parametrized partial differential equations (PDEs) to allow for efficient evaluation of parametrized models in many-query and real-time settings. We use the Reduced Basis model order reduction technique to generate a low order model of an electromagnetic system governed by time-harmonic Maxwell's equations. The reduced order model then makes it feasible to analyze the uncertainty by a Monte Carlo Simulation. Stochastic Collocation is employed as a second technique to estimate the statistics. In particular the combination of model order reduction and Stochastic Collocation allows low computation times.

Key words: Electromagnetic analysis, Maxwell equations, Reduced Basis Method, Reduced Order Systems, Uncertainty Quantification

1 Application Model

As an example model, we consider the coplanar waveguide, shown in Fig. 1. The model setup is contained in a shielded box with perfect electric conducting (PEC) boundary. We consider three perfectly conducting striplines as shown in the geometry. The system is excited at a discrete port and the output is taken at a discrete port on the opposite end of the middle stripline. These discrete ports are used to model input and output currents/voltages.

We are interested in parameter studies of the input-output behavior of electromagnetic models. Therefore, we need to compute the electromagnetic field induced by the applied current. We simulate Maxwell's equations in the second order time-harmonic formulation

$$\nabla \times \mu^{-1} \nabla \times E + j\omega\sigma E - \omega^2 \epsilon E = -j\omega J, \quad (1)$$

and solve for the electric field E. The equation is discretized with Nédélec finite elements (see [1]), over the entire shielded box as the computational domain. The parameter vector is denoted by $\nu \in \mathcal{D} \subset \mathbb{R}^p$, such that $E(\nu)$ is the parameter-dependent electric field solution.

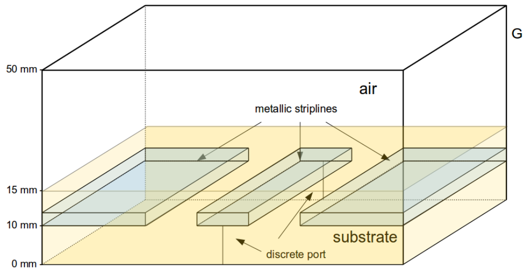

Figure 1: Geometry of a coplanar waveguide.

2 Reduced Basis Model Reduction

Model Order Reduction (MOR) allows to significantly reduce the computational time required for parameter studies. MOR substitutes the large-scale model by a model of low order, which approximates the transfer behavior. The aim of the Reduced Basis Method (RBM) is to determine a low order space X_N of dimension N, which approximates the parametric manifold $M^\nu = \{E(\nu) | \nu \in \mathcal{D}\}$ well. Assuming sufficient smoothness of M^ν, a space X_N can be determined, such that projecting the variational form onto X_N gives good approximations $E_N(\nu)$ to $E(\nu)$. The space X_N is spanned by snapshots of the field solutions for a discrete set of parameter realizations. The snapshot locations are chosen in a greedy process using a rigorous error estimator. The error estimators $\Delta_N(\nu)$, which give rigorous bounds to the approximation error in the $H(\text{curl})$ norm:

$$\|E(\nu) - E_N(\nu)\|_{H(\text{curl})} \leq \Delta_N(\nu), \tag{2}$$

are used to certify the accuracy of the reduced order model. See [2] for more details.

3 Stochastic Collocation

Let $(\Omega, \mathcal{F}, \mathcal{P})$ denote a probability space. Given is a square integrable random variable $Y : \Omega \to \mathbb{R}$ with probability density function f and a function $g : \Gamma \to \mathbb{R}^d$, corresponding to a mapping of realizations of a random variable to the output of the electromagnetic system.

Stochastic collocation computes statistical quantities like the mean by a quadrature rule

$$\mathbb{E}(g(Y)) = \int_\Gamma g(x) f(x) dx \approx \sum_{i=1}^{n} g(\xi_i) w_i, \tag{3}$$

where the realizations ξ_i are the sample points and the weights w_i are determined using the probability density function f. See [3] for more details.

In statistical analysis the expectation and variance of quantities of interest like the transfer behavior under uncertain parameters is computed. For this purpose, stochastic collocation uses a quadrature rule. To further enhance the computation speed of statistical quantities, stochastic collocation is combined with reduced basis model order reduction. This allows to quantify models of a much larger complexity.

References

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