Efficient Simulation for Electrical-Thermal Systems via Multirate-MOR

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Summary. The presented Multirate-MOR scheme exploits the structure of a coupled circuit-thermal system to increase the efficiency of the numerical simulation. The simultaneous use of different step sizes allows the algorithm to adapt to differing dynamical behaviour of the components. We combine this with a model order reduction to obtain smaller subsystems. We will give first numerical results and discuss aspects to be considered in this context.

1 Introduction

We consider an electrical circuit which includes thermal effects. That is, heat is dissipated by certain devices and electrical parameters are temperature dependent. By regularisation of the circuit and semi-discretisation of the thermal part we can achieve a coupled system of ODEs. Here we have to deal with a very different dynamical behaviour of the components of the system: electrical components are changing very fast while components that describe the thermal behaviour change rather slowly. Demanding a high accuracy of the thermal behaviour a fine discretisation of space is needed and we get a large part system with slow dynamic behaviour. To exploit this special structure, we will apply a model order reduction to the thermal part and solve the remaining system of ODEs with a multirate integration scheme.

2 Mathematical Tools

Before modeling the electric-thermal system we introduce the mathematical methods applied in this work.

Mixed-Multirate. Given a system of ODEs with initial values for the unknown $\mathbf{y}$, we partition the system with respect to the dynamic behaviour of the components. For simplicity of notation we assume to identify two part systems with different dynamical behaviour (active: A, and inactive/slow: L):

$$\dot{\mathbf{y}}_A = \mathbf{f}_A(\mathbf{y}_A, \mathbf{y}_L) \quad \mathbf{y}_A(t_0) = \mathbf{y}_{A,0} \tag{1}$$
$$\dot{\mathbf{y}}_L = \mathbf{f}_L(\mathbf{y}_A, \mathbf{y}_L) \quad \mathbf{y}_L(t_0) = \mathbf{y}_{L,0}. \tag{2}$$

In fact, this can generalised to more than two subsystems. The red terms indicate the respective coupling. Now, multirate methods integrate the slow components with large step sizes H and the active components with a much smaller one h while $h \ll H$. The crucial part is how to realise the coupling between the active and the slow part. We are following the idea of [1]: The mixed-multirate compound-step method. Here the coupling is realized by computing the macro-step and the first micro-step coupled manner. For the remaining micro-steps one can either interpolate the slow part or use a dense-output formula. What makes mixed-multirate schemes so interesting is that we can use different integrators for the compound-step and the remaining micro-steps. The underlying ODE integration schemes are given by a linear implicit 2(3)-ROW-method for the compound-step and a 3(4)-ROW-scheme for the remaining micro-steps. So we can handle at least moderately stiff ODEs. A set of coefficients can be found in [1].

MOR with Balanced Truncation. If we deal with a high dimensional slow part a model order reduction (MOR) will help to improve efficiency of the time integration. We limit ourselves to linear MOR so starting point is a linear dynamical system. In the multirate context of (2) this reads

$$\dot{\mathbf{y}}_L = \mathbf{A}\mathbf{y}_L + \mathbf{B}\mathbf{y}_A, \quad \mathbf{y}_L(t_0) = \mathbf{y}_{L,0},$$
$$\mathbf{y}_L = \mathbf{C}\mathbf{y}_L, \qquad \dim(\mathbf{y}_L) = n \quad (\mathbf{C} = \mathbf{Id}).$$

In a MOR method, rectangular biorthogonal projection matrices $\mathbf{V}_r$, $\mathbf{W}_r$ are computed, such that the dimension r of reduced system matrices $\mathbf{W}_r^T\mathbf{A}\mathbf{V}_r$, $\mathbf{W}_r^T\mathbf{B}$, $\mathbf{C}\mathbf{V}_r$ is significantly smaller than the dimension of the original system ($r \ll n$). While the output of the reduced system $\mathbf{y}_{r,L}$ shall approximate the original output as good as possible. The idea of balanced truncation is now to keep all important states and truncate all states which need a large amount of energy to be reached and to be observed. Truncating states that are difficult to reach and to observe become equivalent if the system is balanced. One gets such a balanced system by solving Lyapunov-Equations and construct a suitable transformation matrix. Balanced truncation offers good and reliable error-bounds (cf. [2]) but can be computationally expensive for very large systems.

3 Modeling and Simulation

We apply the multirate-MOR approach the electric circuit shown in Fig. 1 with thermal dependent and thermal active devices. The example was taken from [3] with some modifications.

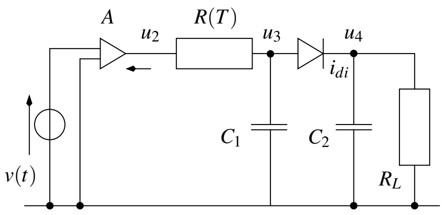

Fig. 1. Circuit Diagram.

Modeling. The resistor $R(T)$ in our example circuit (Fig. 1) is modeled as thermal dependent and thermal active device. On the one hand the resistance is dependent to the temperature and on the other hand the resistor gets heated by the current running through. The diode is modeled as temperature dependent passive element. The circuit is provided by a sinusoidal voltage source for some small time interval. An ODE description of the circuit's voltages can be achieved by nodal analysis using Kirchhoff's law of current for nodes three and four; Joule's law gives the dissipated power at the resistor. In total that reads:

$$C_1\dot{u}_3 = (u_2 - u_3)/R(T) - i_{\mathrm{di}}(u_3 - u_4, T_{\mathrm{di}}) \qquad (3)$$
$$C_2\dot{u}_4 = i_{\mathrm{di}}(u_3 - u_4, T_{\mathrm{di}}) - u_4/R_L \qquad (4)$$
$$\dot{e} = (u_2 - u_1)i_R = (u_2 - u_3)^2/R(T). \qquad (5)$$

The thermal behaviour of the resistor is modeled by a 1D-heat equation with terms for heat conduction, heating due to electric current (Joule's law) and Newton Cooling. To get an ODE description we discretise the spatial variable in the PDE by method of lines and use a finite volume approach. For the inner cells in an equidistant grid we end up with

$$M'_{W,i}\dot{T}_i = \frac{\Lambda}{h^2}(T_{i+1} - 2T_i + T_{i-1}) + P'_{W,i} - \gamma S'_{W,i}(T_i - T_{env}).$$

The boundary cells are similar. The coupling electric to thermal is realised in (5). The other way round we take the solution of the thermal $\mathbf{T}$ part and compute the total resistance $R(\mathbf{T})$. We interpret the temperature of the last cell to be the diode's temperature.

Simulation and Results. To apply the presented multirate-MOR scheme we set the equations of the system of ODEs stemming from the electric behaviour to be the active part and the semi-discretised heat equation to be the slow part. This is a quite natural choice. The description of the thermal behaviour is a priori not linear. To be able to apply a linear MOR method we linearise the thermal part first. To apply a balanced truncation MOR the system has to be complete observable and reachable (cf. [2]) unfortunately the thermal part is not complete reachable. Here we follow an idea of [2] and consider only the reachable subsystem. We also applied a modal order reduction to compare the MOR schemes. In Fig. 2 the relative error to the full system is shown. As expected we get better results by using the balanced truncation.

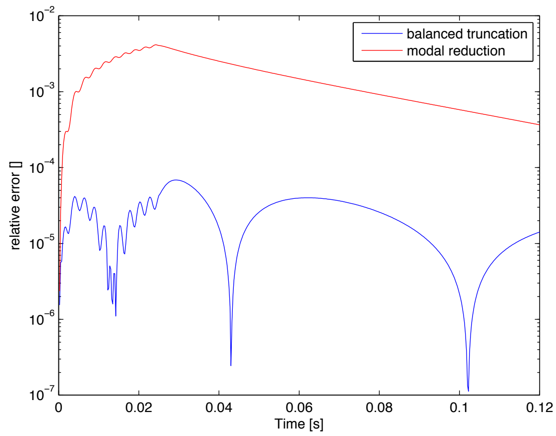

Fig. 2. Relative Error of the Diode's Temperature with different MOR Techniques.

4 Outlook

The applied MOR reduction in a pre-processing step before solving the system with a multirate integrator can only be useful if on the one hand the error due to the MOR is small and on the other hand the computation time decreases significantly. For the first aspect we have to look how sensitive the whole system is with respect to errors in the slow to active interface. For the other aspect we have to consider among others the dimension of the interface. We will present first ideas how to approach to these problems.

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