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D2.3 Intermediate Report on Probability distributions fitted to real data of circuits and devices

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Contents

1 Introduction

In industrial applications, a manufacturing process causes variability in produced electronic circuits of devices [4, 9]. The task, which is considered in this report, is to analyse real data from measurements. We assume that the measurement errors are below the true variations, i.e., these errors are not taken into account.
The strategy can be summarized by two steps:

(i) We check if the data approximatively do result from some random process.

(ii) If the answer in step (i) is positive, then we try to find a probability distribution, which yields a good approximation of the real process.

In particular, spatially distributed measurements are interesting within this context. We consider problems in two space dimensions. Thus several measurements of some quantity are obtained on each spatial element like a wafer, for example. Even if the measured material property is inhomogeneous in space, the spatial profile could be nearly the same for all cases and thus deterministic. The question is if the differences around a mean value are relatively large and if they can be modelled by a random distribution. In case of a positive answer, a model of a random field shall be constructed, where the key figures (expected values, variances, etc.) are obtained from the real data. However, in this intermediate report, we focus on the testing if the data obeys approximately to some probability law, i.e., the step (i) from above.
Three collections of measurements, obtained from different industrial partners in nanoCOPS, are analyzed in this research:

- NXP: bivariate measurements from the currents of an NMOS and of an PMOS device with a large sample size,
- ACC: spatially distributed measurements on wafers with the detection of several quantities per space point,
- ONN: spatially distributed measurements on wafers with the detection of several quantities per space point,

The report is organized as follows. In Section 2, we specify the structuring of measurements for analysis of data as desired for the work in this project. We outline some statistical tests in Section 3, which are applied for the investigation of our data. The main part of the report consists of Section 4, where the data sets are analyzed with repect to underlying probability distributions.

2 Statistical Data Formats

2.1 Introduction

In the near future, large parameter variations are expected within the manufacturing process of electric circuits and electric devices, which make the design and functionality of the end products critical, see [7, 16], for example. Thus the mathematical models have to include the variabilities of the parameters. In the nanoCOPS project, the variations are modelled by random variables for scalar parameters (e.g., capacitances, inductances, resistances, etc.), or random fields for spatial effects (e.g., material parameters, geometries, etc.) to achieve an uncertainty quantification.
The models require the specification of the probability distributions as input. On the one hand, traditional distributions can be chosen (e.g. Gaussian, uniform, etc.). On the other hand, this task will be accomplished by fitting the random distributions to samples, which are obtained by physical measurements of real devices in cooperation with the industrial partners.
From the measurements, key figures like the expected value, the variance, the skew and the kurtosis or other statistical information can be calculated, see [2], for example. Furthermore, correlations of the measurements for different quantities may appear. The numerical simulation of the stochastic models yields results, which allow for approximating the key figures and probability distributions of the outputs.
In the following Sections 2.2 and 2.3, the statistical data formats are described for the two cases of the input data.

2.2 Scalar random parameters

For the scalar quantities, the measurements consist either of a list of scalar numbers or of a list of vectors including the scalars component-wise, i.e.,

$$(m_1^1, \ldots, m_k^1), \quad (m_1^2, \ldots, m_k^2), \ldots, (m_1^l, \ldots, m_k^l)$$

where, simultaneously, for k different quantities, l measurements are done. It should be specified separately which different settings have been chosen to obtain the measurements. For example, (i) each dataset $(m_1, \ldots, m_k)$ can be associated to different electric devices from the same manufacturing process, or, (ii) each dataset $(m_1, \ldots, m_k)$ is taken from the same device at different time points $t_1, \ldots, t_l$ in the magnitude of weeks or years to investigate ageing. In case (i), a permutation of the measurements does not change the statistical properties or key figures. In case (ii), additional specifications of the setting or coordinates play a role and the ordering of the measurements is intrinsic.

2.3 Spatial random fields

For spatial processes, the correlations shall be analyzed. The one-, two- and three-dimensional case is feasible. In the following, the three-dimensional case is described, since the other two settings appear by simply omitting dimensions. Furthermore, just a scalar quantity is considered in each space point, because the extension to vector-valued formats is straightforward.

2.3.1 Structured measurements

In a cuboid, a uniform grid is given, where a measurement is done in each grid point. Hence the data are written in the form

$$(x_i, y_j, z_k, m_{ijk}) \text{ for } i = 1, \ldots, n_1,\ j = 1, \ldots, n_2,\ k = 1, \ldots, n_3,$$

where m_{ijk} represents the measurement of the quantity at the space point (x_i, y_j, z_k). Alternatively, the measurements can be arranged in a three-dimensional field of numbers and the information of the space points is stored separately.
Often the spatial domain is designed using a uniform grid of points or cells, whereas only a few measurements are done in the domain. In this case, the above data format is still feasible, where cells without a measurement just include an empty entry for m_{ijk}. This data format often agrees to the structure required for procedures in graphical illustration (post-processing).

2.3.2 Unstructured measurements

Now the position of the space points for the measurements are arbitrary. Thus the data are written as

$$(x_i, y_i, z_i, m_i) \text{ for } i = 1, \ldots, l,$$

with m_i being again the measurement at space point (x_i, y_i, z_i). In this case, an arrangement of the measurements in a three-dimensional field of numbers does not make sense.

3 Statistical tools description for data analysis

Amongst all available data delivered by our partners we decided to choose one representative physical quantity from every measurement category mainly in order to investigate following items:

- the normality tests of data under consideration;
- the spatial in/dependences of the data;
- the homogeneity/anisotropy of the material.

Answering these questions allows us to take decision how to model the material in our models and how we can handle uncertainties related to parameter identification in our model (which kind of distributions should we take into account during simulation, how this model could be calibrated then, etc.).
For the moment we restrict ourselves to several Goodness-of-Fit tests [1]. These tests indicate if a certain distribution occurs. Graphical tools, like QQ-plots, normal probability plots and kernel density estimators [11] (available in the Matlab Statistical Toolbox) can also reveal why a certain distribution does not apply.

3.1 Description of tests under consideration

The Shapiro-Wilk test [14] for the verification of the hypothesis about the normal distribution and the Levene's test [6] for the verification of variances homogeneity will be applied. If both tests return positive answers, the ANOVA (Analysis of Variances) test [12] is applied. Otherwise, a special ANOVA test, the so-called the Kruskal-Willis test [3] (which is non-parametric, and distribution-free in assumption), is used. In both cases, the desired significance level, or α-level, used $\alpha = 0.05$.

3.2 Shapiro-Wilk test

The Shapiro-Wilk test [14] verifies the null hypothesis H_0 versus the alternative H_1 to check whether a sample x_1, ..., x_n belongs to a normally distributed population. The construction of the statistical test value W is as follows :

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

where : $x_{(i)}$ denotes i-th order statistic, that is the i-th smallest number in the sample, $\bar{x}$ is the mean value, while the constraints a_i are defined by :

$$(a_1, ..., a_n) = \frac{m^\top V^{-1}}{\left(m^\top V^{-1} V^{-1} m\right)^{1/2}}, \quad \text{with } m = (m_1, ..., m_n)^\top,$$

in which m_1, ..., m_n are the mean values of the order statistics of independent and identically distributed random variables, which are sampled from the standard normal distribution, while V denotes the covariance matrix of those order statistics (which is symmetric, hence $V^\top = V$). The null hypothesis is rejected if the statistical test value W is below a predetermined threshold.

3.3 Homogeneity variance test - the Levene's test

In statistics, Levene's test [6] is used to assess the variances equality in the case when a variable is calculated for two, or more, groups. Some common statistical procedures like the ANOVA tests (Analysis of Variances), for example, requires that variances, of the populations from which different samples are drawn, will be equal. Levene's test assesses this assumption. More precisely, it checks the null hypothesis that the population variances are equal, the so-called homogeneity of variance or homoscedasticity assumption. If the resulting p-value of Levene's test is less than some critical value α (typically α = 0.05), the null hypothesis of equal variances is rejected because there is a difference between the variances in the population. Thus, the p-value is the probability of receiving a test statistic result at least as extreme as the one that was actually observed, under assumption that the null hypothesis is true [6]. As in the case of the Shapiro-Wilk test (Section 3.2), a statistical test value W is calculated. The null hypothesis is rejected if the statistical test value W is below a predetermined threshold. The definition of W needs some preparation.

We assume k groups of samples Y_{ij}, where i denotes the group number and j the j-th sample within group i. Here $j \leq N_i$, where N_i is the number of cases in the i-th group. From these we derive

$$Z_{ij} = \begin{cases} \left|Y_{ij} - \bar{Y}_{i\cdot}\right|, & \text{when using the mean of the } i\text{-th group, } \bar{Y}_{i\cdot} \\ \left|Y_{ij} - \tilde{Y}_{i\cdot}\right|, & \text{when using the median of the } i\text{-th group, } \tilde{Y}_{i\cdot}. \end{cases}$$

The overall mean value $Z_{\cdot\cdot}$ of all Z_{ij}, and the i-th mean $Z_{i\cdot}$ of the Z_{ij} for the i-th group, are defined as follows

$$Z_{\cdot\cdot} = \frac{1}{N}\sum_{i=1}^{k}\sum_{j=1}^{N_i} Z_{ij} \text{ and } Z_{i\cdot} = \frac{1}{N_i}\sum_{j=1}^{N_i} Z_{ij}.$$

Now the definition of the statistical test quantity W can be made

$$W = \frac{(N-k)\sum_{i=1}^{k} N_i \left(Z_{i\cdot} - Z_{\cdot\cdot}\right)^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{N_i}\left(Z_{ij} - Z_{i\cdot}\right)^2}.$$

As said before, the null hypothesis is rejected if the statistical test value W is below a predetermined threshold.

3.4 The Analysis Of Variance ANOVA

The one-way analysis of variance, the so-called one-way ANOVA [12], is a method for comparing means of two or more samples using the $F-$ distribution. Specifically, the ANOVA verifies the null hypothesis that samples in two or more groups come from populations with the same mean values versus alternative hypothesis that they are drawn from populations with different means. For this purpose, two estimates are made of the population variance which are given below. The results of a one-way ANOVA can be considered as reliable if the following assumptions are fulfilled:

- the response variable residuals are normally distributed;
- the samples are independent;
- the variances of the populations are equal;
- the responses for a given group are independent and are identically distributed normal random variables [13].

The system of hypothesis under consideration is as follows :

$$\begin{aligned} &\text{The null hypothesis:} && H_0 : \mu_1 = \mu_2 = ... = \mu_r, \\ &\text{The alternative hypothesis:} && H_1 : \mu_i \neq \mu_j, \text{ for some } i \neq j. \end{aligned}$$

Finally, the ANOVA produces an F-statistic, the ratio of the variance calculated among the means to the variance within the samples :

$$F = \frac{MSTR}{MSE},$$

where particular components are defined by:

$$MSTR = \frac{1}{r-1} \sum_{i=1}^{r} n_i \left(\bar{x}_i - \hat{x}\right)^2 \text{ and } MSE = \frac{1}{r-1} \sum_{i=1}^{r} \sum_{j=1}^{n_i} \left(x_{ij} - \bar{x}_i\right)^2.$$

Here, $\bar{x}_i$ denotes the arithmetic mean of the *i*th group, while $\hat{x}$ refers to the arithmetic mean of all the observations including all the r trials.
If the group means are drawn from populations with the same mean values, the variance between the group means should be lower than the variance of the samples (following the central limit theorem). Otherwise it is concluded, that the samples are drawn from populations with different mean values.
A non-parametric alternative to this test, like Kruskal-Wallis one-way analysis of variance (see next Section), ought to be used when data does not meet the above-mentioned assumptions.

3.5 Kruskal-Wallis test

The Kruskal-Wallis one-way analysis of variance by ranks is a non-parametric equivalent of the one-way analysis of variance (ANOVA), which does not assume a normal distribution of the residuals. It is called "distribution-free". Thus, this technique is applied for testing whether samples originate from the same distribution. When the Kruskal-Wallis test leads to significant results, it means that at least one of the samples follows from a different population then for the others. However, the test does not give the answer for following questions, like where or how many differences actually occur [3]. The test statistic is constructed as follows:

$$T = \frac{12}{n\left(n+1\right)} \sum_{i=1}^{k} n_i \left(\bar{R}_i - \frac{n+1}{2}\right)^2,$$

where $n \sum_{i=1}^{k} n_i$, with the trial statistics devided into k groups with numbers n_1, n_2, ..., n_k. The quantity $\bar{R}_i$ is defined as:

$$\bar{R}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} R_{ij}.$$

Here R_{ij} is the rank (among all observations) of observation j from group i.
The statistic T is the discrepancy of the mean for the sampled ranks from the mean value from the overall rank, which is of (n+1)/2. The test assumes an identically shaped and scaled distribution for each group, except for any difference in medians. It also can be applied for examining groups that have unequal size [3].

3.6 Mardia test

Given l different measurements of k properties

$$(m_1^j, m_2^j, \ldots, m_k^j) \quad \text{for} \;\; j = 1, \ldots, l,$$

we will check if the tuples result from a single k-dimensional normal distribution using Mardia's test [10]. Therein, the expected value and the covariance matrix are arbitrary, i.e., the type of the distribution is checked qualitatively. In this statistical method, the skewness and the kurtosis

$$\mathrm{SKEWNESS} := \mathrm{E}\left[\left(\frac{X-\mu}{\sigma}\right)^3\right], \qquad \mathrm{KURTOSIS} := \frac{\mathrm{E}\left[(X-\mu)^4\right]}{\sigma^4},$$

which are associated to the third and fourth moment of the random distribution, are considered. Consequently, the discrete skewness and discrete kurtosis are calculated and analyzed. Two different criteria are involved in this test, where the result can be different for skewness and kurtosis. However, if both statistics yield the same result, then the conclusions are more reliable. We apply the implentation from [15] to perform this method.

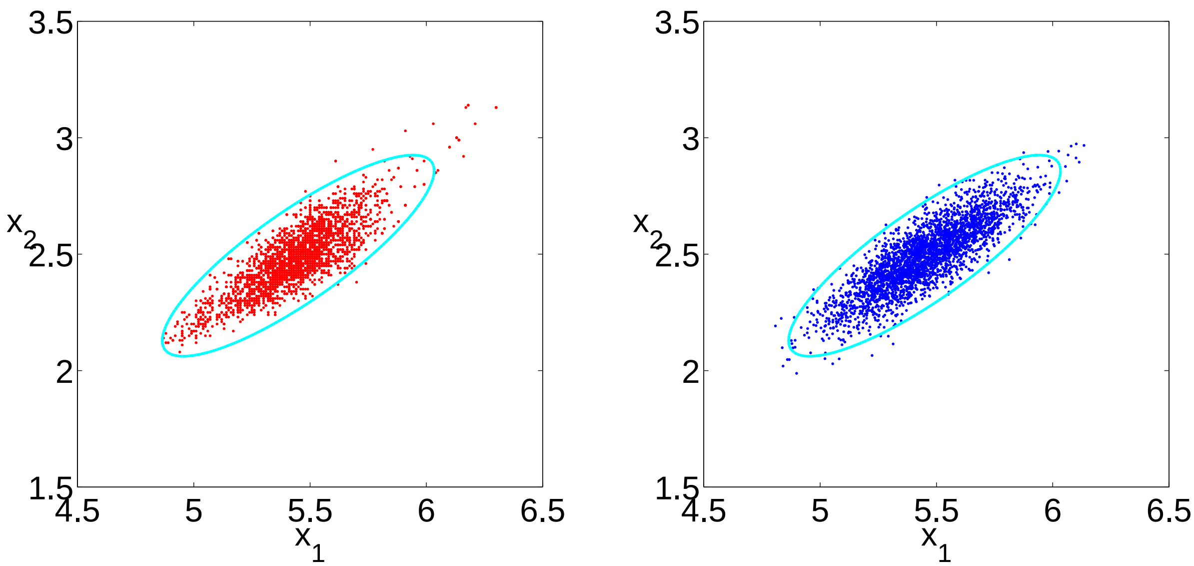
4 Industrial Data and Results

4.1 NXP Data

The industrial partner NXP provided measurements on the currents of an NMOS and of an PMOS device for a certain technology:

131219__NXP__Data.dat

The data consist of $n = 2715$ bivariate measurements. Hence the main task is to analyse the correlations between the two measured properties. Fig. 1 (left) shows these measurements. x_1 and x_2 are the currents of an NMOS and of a PMOS, resp. Also a 3σ ellipse is shown (eq of the mean and the cov matrix). Multivariate normal distributions are a special class of ”elliptically contoured distributions”.



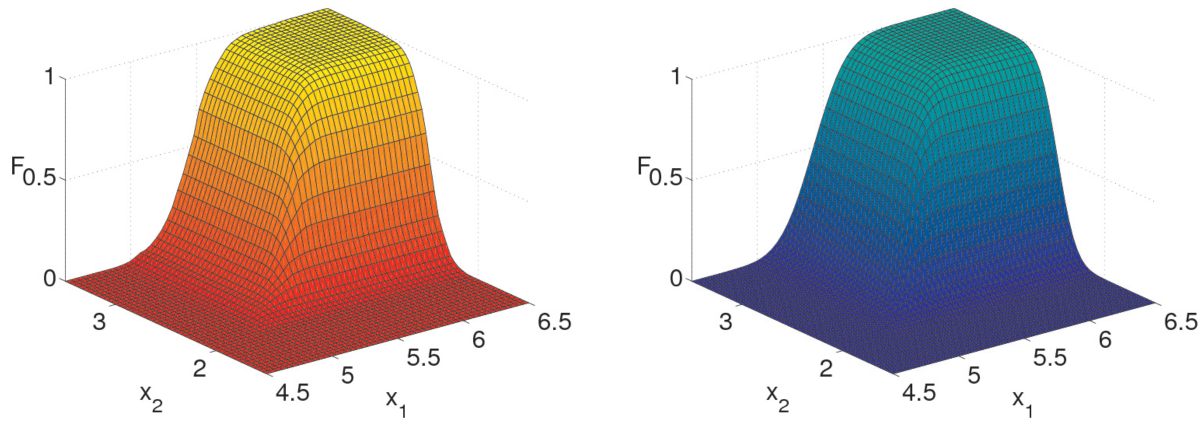
(a) Measurements (b) Random samples

Figure 1: Measurements in the NXP example (left) and samples from an approximating normal distribution (right). x_1 and x_2 are the currents of an NMOS and of a PMOS, resp. In both cases also a 3σ ellipse is shown (eq of the mean and the cov matrix).

Using the discrete data, the mean μ and the covariance matrix Σ are computed to

$$\mu = \begin{pmatrix} 5.4488 \\ 2.4936 \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} 0.0380 & 0.0237 \\ 0.0237 & 0.0207 \end{pmatrix}.$$

We define a normal distribution using this mean and covariance matrix to approximate the underlying process, where this choice is optimal in some sense. Fig. 1 (right) shows n realisations of this normal distribution, where the samples follow from pseudo random numbers. Furthermore, the empirical distribution function of the discrete data as well as the cumulative distribution function of the approximating bivariate normal distribution are illustrated by Fig. 2. The error of the approximation on the level of the distribution function is depicted in Fig. 3. We observe a good agreement.
We perform a statistic test to check the null hypothesis that our data results from a bivariate normal distribution. Mardia’s method [10] is applied to the data. It follows



(a) empirical distribution fcn.

(b) cumulative distribution fcn.

Figure 2: Empirical distribution function of the measurements (left) and the cumulative distribution function of approximating normal distribution (right).

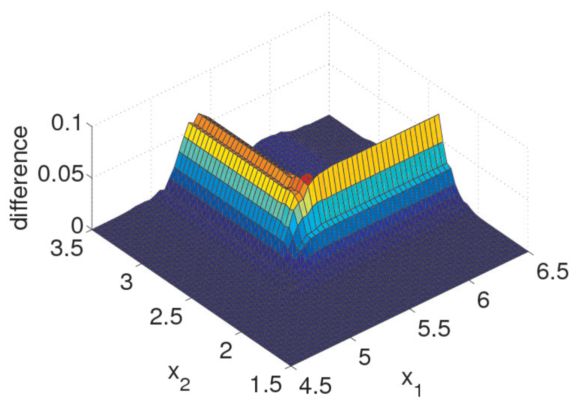


Figure 3: Difference between the empirical distribution function and the approximating cumulative distribution function. Clearly there is a difference in the tails.

that the p-values for both, skewness (0.2200) and kurtosis (8.9310), are smaller than $\alpha = 0.0001$. Thus we reject the null hypothesis for both criteria on a relatively small significance level.

Conclusions : The measured data do not result from a bivariate normal distribution. Nevertheless a fitted bivariate normal distribution already provides an acceptable approximation. To increase the accuracy, alternative methods for approximation of random distributions have to be checked.

Note that, in general, currents of NMOS and PMOS devices are themselves functions of input parameters. Even if these input parameters are subject to variations according to a normal distribution, it is not guaranteed that the resulting currents show a normal distribution.

Outlook : An improvement could be obtained by Fleishman's power method, see [5, 8]. This approach includes the above fitting as a special case. Thus its approximation must be at least as good as the straightforward bivariate normal distribution, or better.

4.2 ACCO Data

The industrial SME partner ACCO did provide technological data as well as measurements of the AC12050 ACCO product. Two excel files were used for this:

1. Technological__data.xlsx: ACCO used three different technologies. Their physical and electrical descriptions have been delivered. It includes suppliers data: typical device parameters and technological/material (backend) description with process variation: (BT, IPD, SI).

2. Measurements__AC12050__201312.xlsx: ACCO did provide PCM (Process Control Monitor) measurements on different sites of different wafers and lots. PCM concern backend characteristics, electrical device parameters, resistances, capacitances The first database given by ACCO concerned SI wafers only, on 2 different lots, for each lot on 12 different wafers and for each wafer on 5 different wafer sites. Measurements concern: Backend, Active devices, resistances, capacitances parameters.

4.2.1 General description and format of ACCO Data

In general, the data delivered by our project partner ACCO are divided into several measurement categories: backend data, Thick NFET, Thick PFET, PFET, NFET, JFET, MIM Capacitance, Resistor, Diffusion resistances. Each category, besides JFET, includes measurement data performed for 24 pieces of wafers in five measurement points, described in a 2D Cartesian coordinate system. In the case of JFET, the measurement was carried out in five different points, specified in the same way as previous data, but for 48 wafers. Moreover, within each category one can find the rough data description with physical quantities under consideration, for example, its units, target name, measurements, code, specification high and low range, etc. For the statistical analysis, the commercial software Statistica version 10.0 MR1 has been used [17].

4.2.2 Results of the statistical analysis

In this section, only one representative data from the measurement category of the ACCO data are taken into account.

- Backend: E2 resistivity
- Thick NFET: ID$_{sat}$
- Thick PFET: Vt$_{Linear}$
- PFET: Vt$_{sat}$
- MIM Capacitance: HIK MIM Density

Furthermore, in order to consider our focus points in checking: (i) the normality distribution of data under consideration, (ii) the spatial in/dependences of these data, (iii) the homogeneity/anisotropy of the underlying material, the ANOVA/the Kruskal-Wallis tests are used. The main results are presented in the form of tables. More precisely, we will follow the procedure that is described above. Thus, first the assumption on the normal distribution of data will be verified using the Shapiro-Wilk test. If the obtained answer is positive, then the homogeneity variance test is applied. Next, depending on the results, the ANOVA or the Kruskal-Wallis test will be used for finding the answers for the questions : (i)-(iii).

- Measurements__AC12050__201312/ Backend/ **E2 resistivity**

Table 1. Descriptive statistics: for all data

Without grouping descriptive statistics (base total)								
	Numbers	**Mean**	**Median**	**Min.**	**Max.**	**f. quart. Q_1**	**third quart. Q_3**	**Stand. Dev.**
m	120	0,103386	0,103893	0,098324	0,106089	0,102452	0,104680	0,001814

Table 2.Without grouping descriptive statistics, cardinality table: Shapiro-Wilk: W = 0,91625, **p =0,000**

Without grouping descriptive statistics (base total)						
	Number	**Cumulative numb.**	**Percentage - valid**	**Cumul. % - valid**	**% of the total - Cases**	**Cumulative %**
,0970000<x<=,0980000	0	0	0,00000	0,0000	0,00000	0,0000
,0980000<x<=,0990000	3	3	2,50000	2,5000	2,50000	2,5000
,0990000<x<=,1000000	4	7	3,33333	5,8333	3,33333	5,8333
,1000000<x<=,1010000	9	16	7,50000	13,3333	7,50000	13,3333
,1010000<x<=,1020000	12	28	10,00000	23,3333	10,00000	23,3333
,1020000<x<=,1030000	9	37	7,50000	30,8333	7,50000	30,8333
,1030000<x<=,1040000	28	65	23,33333	54,1667	23,33333	54,1667
,1040000<x<=,1050000	36	101	30,00000	84,1667	30,00000	84,1667
,1050000<x<=,1060000	17	118	14,16667	98,3333	14,16667	98,3333
,1060000<x<=,1070000	2	120	1,66667	100,0000	1,66667	100,0000
Lack	0	120	0,00000		0,00000	100,0000

Table 3. The value of 'z' for the multiple comparisons; E2_resistivity (base total) Kruskal-Wallis Test: H (4, N= 120) = 82,38518, **p =,0000**

	1 - R:18,938	**2 - R:39,125**	**3 - R:63,708**	**4 - R:81,021**	**5 - R:99,708**
1		2,010391	4,458545	6,182626	8,043638
2	2,010391		2,448154	4,172235	6,033247
3	4,458545	2,448154		1,724081	3,585093
4	6,182626	4,172235	1,724081		1,861012
5	8,043638	6,033247	3,585093	1,861012	

Table 4. Value ' p' for the multiple comparison E2_resistivity (base total) Kruskal-Wallis Test: H (4, N= 120) =82,38518, **p =,0000**

	1 - R:18,938	**2 - R:39,125**	**3 - R:63,708**	**4 - R:81,021**	**5 - R:99,708**
1		0,443898	0,000083	0,000000	0,000000
2	0,443898		0,143590	0,000302	0,000000
3	0,000083	0,143590		0,846932	0,003370
4	0,000000	0,000302	0,846932		0,627425
5	0,000000	0,000000	0,003370	0,627425	

Remarks : A test for variance is not required due to the result of the Shapiro-Wilk test for the normal distribution verification. Thus, the nonparametric Kruskal-Wallis test is used because of the fact, that the test on the normality for the investigated quantities failed. In this case, the green color denotes a significant difference between particular groups, which means that the null hypothesis is rejected and, in consequence, it is concluded, that the data tested are not from a normally distributed population.

Conclusions : (i) the analyzed material considering the E2 resistivity is not homogeneous and it might be anisotropic because the means calculated for the particular groups are statistically significant (the green color), (ii) the analyzed data for every measurement point (five groups) are spatially independent, which comes directly from the first conclusion, (iii) in consequence, the measurement data do not belong to the same population and cannot be described by the same normal distribution. The median distribution, together with quartiles Q_1 and Q_3, is presented on Fig. 4.

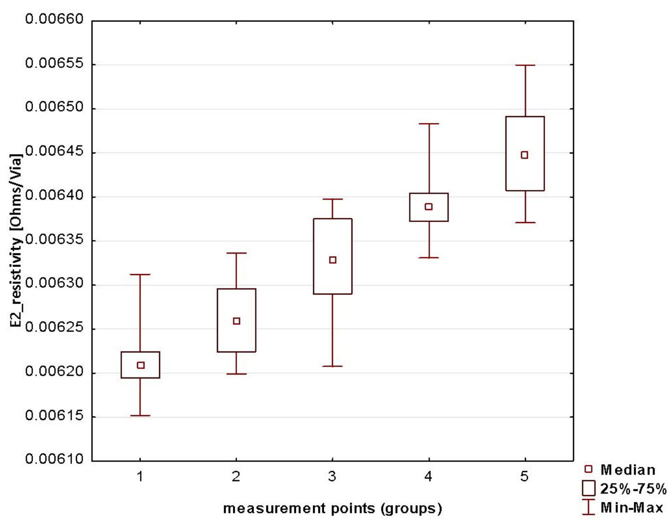


Figure 4: The median distribution together with quartiles Q_1 and Q_3.

- Measurements__AC12050__201312/ Thick NFET/ **Thick NFET Id__sat**

Table 5. Descriptive statistics: for all data

Without grouping descriptive statistics (base total)								
	Numbers	**Mean**	**Median**	**Min.**	**Max.**	**f. quart. Q_1**	**third quart. Q_3**	**Stand. Dev.**
m	120	578,2570	578,7220	568,3820	587,3160	575,5660	580,9370	3,908859

Table 6. Without grouping descriptive statistics, cardinality table Shapiro-Wilk W=0,98848, **p=,40875**

Without grouping descriptive statistics (base total)						
	Number	**Cumulative numb.**	**Percentage - valid**	**Cumul. % - valid**	**% of the total - Cases**	**Cumulative %**
565,0000<x<=570,0000	3	3	2,50000	2,5000	2,47934	2,4793
570,0000<x<=575,0000	19	22	15,83333	18,3333	15,70248	18,1818
575,0000<x<=580,0000	63	85	52,50000	70,8333	52,06612	70,2479
580,0000<x<=585,0000	28	113	23,33333	94,1667	23,14050	93,3884
585,0000<x<=590,0000	7	120	5,83333	100,0000	5,78512	99,1736
Lack	1	121	0,83333		0,82645	100,0000

Table 7. the Levene's test on the variances equality

the Levene's test on the variances equality (base total) **p < ,05000**								
	SS - Efekt	**df - Efekt**	**MS - Efekt**	**SS - Error**	**df - Error**	**MS -Error**	**F**	**P**
Thick NFET Id_sat	4,231084	4	1,057771	268,4725	115	2,334543	0,453095	0,769957

Table 8. The value of 'z' for the multiple comparisons; Thick NFET Id_sat (base total) Test Kruskal-Wallis: H (4, N= 120) =67,95092, **p =,0000**

	1 - R:22,125	**2 - R:43,042**	**3 - R:62,271**	**4 - R:77,979**	**5 - R:97,083**
1		2,083005	3,997960	5,562289	7,464795
2	2,083005		1,914954	3,479283	5,381789
3	3,997960	1,914954		1,564329	3,466835
4	5,562289	3,479283	1,564329		1,902506
5	7,464795	5,381789	3,466835	1,902506	

Table 9. Value ' p' for the multiple comparison ; Thick NFET Id_sat (base total) Test Kruskal-Wallis: H (4, N= 120) =67,95092, **p =,0000**

	1 - R:22,125	**2 - R:43,042**	**3 - R:62,271**	**4 - R:77,979**	**5 - R:97,083**
1		0,372507	0,000639	0,000000	0,000000
2	0,372507		0,554983	0,005028	0,000001
3	0,000639	0,554983		1,000000	0,005266
4	0,000000	0,005028	1,000000		0,571050
5	0,000000	0,000001	0,005266	0,571050	

Remarks : Here, a test for variance is required due to the result of the test for the normal distribution. However, the null hypothesis in the Levene's test has been rejected. Hence, the nonparametric Kruskal-Wallis test has been used. Also in this case, the green color denotes that the null hypothesis is rejected.

Conclusions : (i) the analyzed material considering Thick NFET Id__sat is not homogeneous because the means calculated for the particular groups are statistically significant (the green color), (ii) the analyzed data for every measurement point (five groups) are spatialy independent, which results directly from the first conclusion, (iii) in consequence, the measurement data do not belong to the same population and cannot be described by the same normal distribution. The median distribution, together with quartiles Q_1 and

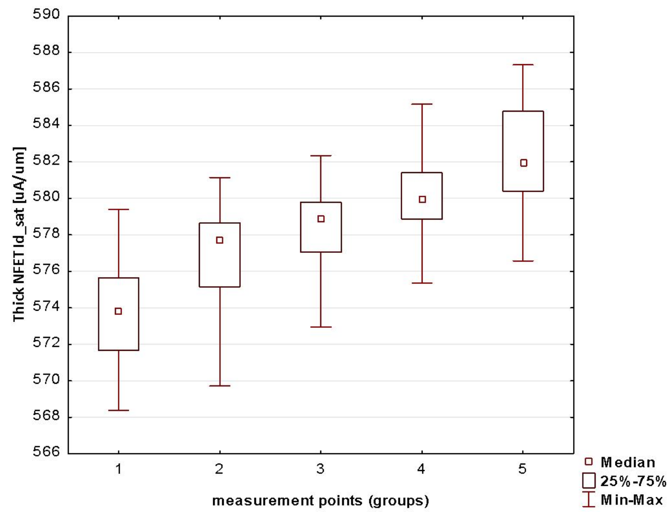


Figure 5: The median distribution together with quartiles Q_1 and Q_3.

Q_3, is shown on Fig. 5.

- Measurements__AC12050__201312/ Thick PFET/ **Thick PFET Vt__linear**

Table 10. Descriptive statistics: for all data

Without grouping descriptive statistics (base total)							
	Numbers	**Median**	**Min.**	**Max.**	**F. quart. Q_1**	**Fhird quart. Q_3**	**Stand. Dev.**
m	120	-0,718050	-0,730596	-0,703909	-0,721936	-0,714811	0,005141

Table 11.Without grouping descriptive statistics, cardinality table: Shapiro-Wilk W=,98058, **p=,08069**

Without grouping descriptive statistics (base total)						
	Number	**Cumulative numb.**	**Percentage - valid**	**Cumul. % - valid**	**% of the total - Cases**	**Cumulative %**
-,740000<x<=-,735000	0	0	0,00000	0,0000	0,00000	0,0000
-,735000<x<=-,730000	1	1	0,83333	0,8333	0,82645	0,8264
-,730000<x<=-,725000	6	7	5,00000	5,8333	4,95868	5,7851
-,725000<x<=-,720000	42	49	35,00000	40,8333	34,71074	40,4959
-,720000<x<=-,715000	40	89	33,33333	74,1667	33,05785	73,5537
-,715000<x<=-,710000	22	111	18,33333	92,5000	18,18182	91,7355
-,710000<x<=-,705000	8	119	6,66667	99,1667	6,61157	98,3471
-,705000<x<=-,700000	1	120	0,83333	100,0000	0,82645	99,1736
Lack	1	121	0,83333		0,82645	100,0000

Table 12. The value of 'z' for the multiple comparisons; Thick PFET Vt_linear (base total) Kruskal-Wallis Test: H (4, N= 120) =84,70178, p =,0000

	1 - R:21,250	2 - R:37,854	3 - R:57,896	4 - R:84,708	5 - R:100,79
1		1,653541	3,649409	6,319556	7,921230
2	1,653541		1,995868	4,666015	6,267689
3	3,649409	1,995868		2,670147	4,271821
4	6,319556	4,666015	2,670147		1,601674
5	7,921230	6,267689	4,271821	1,601674	

Table 13. Value ' p' for the multiple comparison; Thick PFET Vt_linear (base total) Kruskal-Wallis Test: H (4, N= 120) =84,70178; p =,0000

	1 - R:21,250	2 - R:37,854	3 - R:57,896	4 - R:84,708	5 - R:100,79
1		0,982208	0,002628	0,000000	0,000000
2	0,982208		0,459483	0,000031	0,000000
3	0,002628	0,459483		0,075818	0,000194
4	0,000000	0,000031	0,075818		1,000000
5	0,000000	0,000000	0,000194	1,000000	

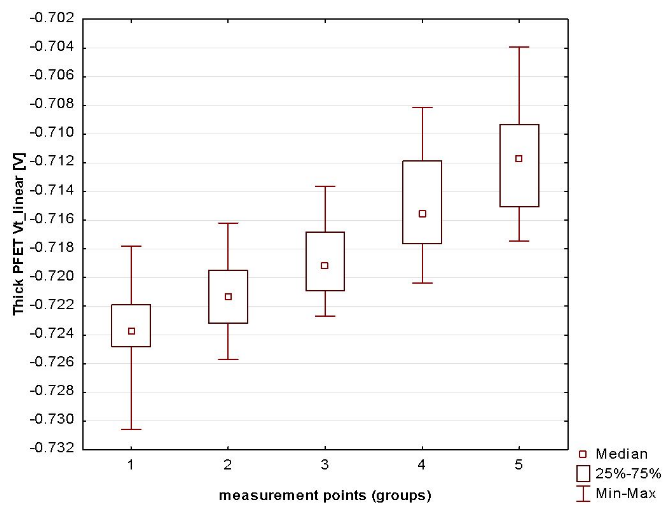


Figure 6: The median distribution together with quartiles Q_1 and Q_3.

Remarks : The test for variance is not required because of the result of the test for the normal distribution, thus the nonparametric Kruskal-Wallis test is used. In this case, the green color denotes a significant difference between particular groups, which implies that the data tested are not from a normally distributed population.

Conclusions : (i) the analyzed material for Thick PFET Vt__linear is not homogeneous because the means calculated for particular groups are statistically significant (the green color), (ii) the analyzed data for every measurement point (five groups) are spatially independent, which results directly from the first conclusion, (iii) as a result, the measurement data do not belong to the same population. The median distribution, together with quartiles Q_1 and Q_3, is shown in Fig. 6.

- Measurements__AC12050__201312/ PFET/ **PFET Vt__sat**

Table 14. Descriptive statistics: for all data

Without grouping descriptive statistics (base total)					
	Numbers	**Mean**	**Min.**	**Max,**	**Stand. Dev.**
PFET Vt_sat	120	-0,362597	-0,384510	-0,328260	0,010961

Table 15.Without grouping descriptive statistics, cardinality table: Shapiro-Wilk; W=,98516; **p=,21245**

Without grouping descriptive statistics (base total)						
	Number	**Cumulative numb.**	**Percentage - valid**	**Cumul. % - valid**	**% of the total - Cases**	**Cumulative %**
-,400000<x<=- ,390000	0	0	0,00000	0,0000	0,00000	0,0000
-,390000<x<=- ,380000	6	6	5,00000	5,0000	4,95868	4,9587
-,380000<x<=- ,370000	28	34	23,33333	28,3333	23,14050	28,0992
-,370000<x<=- ,360000	37	71	30,83333	59,1667	30,57851	58,6777
-,360000<x<=- ,350000	34	105	28,33333	87,5000	28,09917	86,7769
-,350000<x<=- ,340000	13	118	10,83333	98,3333	10,74380	97,5207
-,340000<x<=- ,330000	1	119	0,83333	99,1667	0,82645	98,3471
-,330000<x<=- ,320000	1	120	0,83333	100,0000	0,82645	99,1736
Braki	1	121	0,83333		0,82645	100,0000

Table 16. The value of 'z' for the multiple comparisons; PFET Vt_sat (base total) Kruskal-Wallis Test: H (4, N= 120) =67,71683; p =,0000

	1 - R:26,188	**2 - R:41,000**	**3 - R:58,438**	**4 - R:76,375**	**5 - R:100,50**
1		1,475116	3,211646	4,997968	7,400479
2	1,475116		1,736529	3,522852	5,925362
3	3,211646	1,736529		1,786322	4,188833
4	4,997968	3,522852	1,786322		2,402510
5	7,400479	5,925362	4,188833	2,402510	

Table 17. Value ‘ p' for the multiple comparison; PFET Vt_sat (base total) Kruskal-Wallis Test: H (4, N= 120) =67,71683; **p =,0000**

	1 - R:26,188	**2 - R:41,000**	**3 - R:58,438**	**4 - R:76,375**	**5 - R:100,50**
1		1,000000	0,013198	0,000006	0,000000
2	1,000000		0,824703	0,004269	0,000000
3	0,013198	0,824703		0,740471	0,000280
4	0,000006	0,004269	0,740471		0,162830
5	0,000000	0,000000	0,000280	0,162830	

Remarks : A test for the variance is not required due to the result of the Shapiro-Wilk test, and in cosequence, also in this case, the nonparametric Kruskal-Wallis test is applied. The green color denotes a significant difference between particular groups.
Conclusions : (i) the analyzed material for PFET Vt__sat is not homogeneous and might be anisotropic, which resulted in the Kruskal-Wallis test (the means calculated for particular groups are statistically significant - the green color), (ii) the analyzed data for every measurement point (five groups) are spatially independent, (iii) in consequence, the measurement data do not belong to the same population and cannot be described by the same normal distribution. The median distribution, together with quartiles Q_1 and Q_3, is shown in Fig. 7.

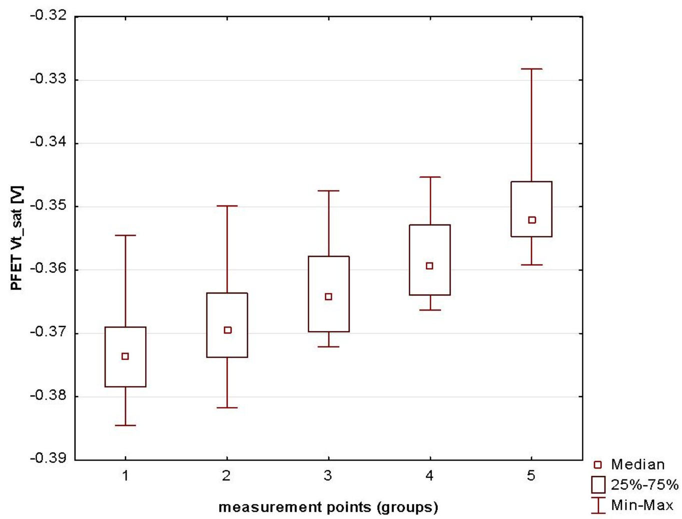


Figure 7: The median distribution together with quartiles Q_1 and Q_3.

- Measurements__AC12050__201312/ MIM Capacitance/ **HiK MIM Densityt**

Table 18. Descriptive statistics: for all data

Without grouping descriptive statistics (base total)							
	Numbers	**Mean**	**Min.**	**Max.**	**F. quart. Q_1**	**Third quart. Q_3**	**Stand. Dev.**
m	120	4,038023	3,935287	4,121847	3,997468	4,080382	0,047640

Table 19.Without grouping descriptive statistics, cardinality table: Shapiro-Wilk W=,95193, **p=,00030**

Without grouping descriptive statistics (base total)						
	Number	**Cumulative numb.**	**Percentage - valid**	**Cumul. % - valid**	**% of the total - Cases**	**Cumulative %**
3,900000<x<=3,950000	2	2	1,66667	1,6667	1,65289	1,6529
3,950000<x<=4,000000	32	34	26,66667	28,3333	26,44628	28,0992
4,000000<x<=4,050000	28	62	23,33333	51,6667	23,14050	51,2397
4,050000<x<=4,100000	50	112	41,66667	93,3333	41,32231	92,5620
4,100000<x<=4,150000	8	120	6,66667	100,0000	6,61157	99,1736
Braki	1	121	0,83333		0,82645	100,0000

Table 20. The value of 'z' for the multiple comparisons; HiK MIM Density (base total) Kruskal-Wallis Test: H (4, N= 120) =94,86792, p =,0000

	1 - R:17,104	**2 - R:36,708**	**3 - R:61,521**	**4 - R:85,354**	**5 - R:101,81**
1		1,952299	4,423275	6,796739	8,435757
2	1,952299		2,470976	4,844440	6,483458
3	4,423275	2,470976		2,373464	4,012483
4	6,796739	4,844440	2,373464		1,639018
5	8,435757	6,483458	4,012483	1,639018	

	1 - R:17,104	2 - R:36,708	3 - R:61,521	4 - R:85,354	5 - R:101,81
1		0,509027	0,000097	0,000000	0,000000
2	0,509027		0,134745	0,000013	0,000000
3	0,000097	0,134745		0,176221	0,000601
4	0,000000	0,000013	0,176221		1,000000
5	0,000000	0,000000	0,000601	1,000000	

Table 21. Value ' p' for the multiple comparison; HiK MIM Density (base total) Kruskal-Wallis Test: H (4, N= 120) =94,86792; p =,0000

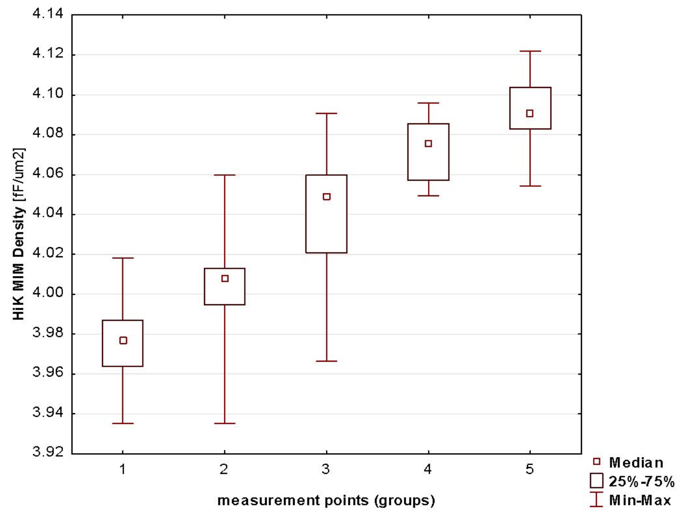


Figure 8: The median distribution together with quartiles Q_1 and Q_3.

Remarks : A test for variance is not required because of the result of the test for the normal distribution, hence, the nonparametric Kruskal-Wallis test is used. Also in this situation, the green color denotes a significant difference between particular groups.

Conclusions : (i) the analyzed material for HiK MIM Density is not homogeneous due to the fact, that the means calculated for particular groups are statistically significant (the green color), (ii) the analyzed data for every measurement point (five groups) are spatially independent, this conclusion results from the first statement, (iii) as a result, the measurement data does not belong to the same population. The median distribution, together with quartiles Q_1 and Q_3, is shown in Fig. 8.

4.2.3 General conclusions

In a next series ACCO will measure all sites of wafers (not just 5 wafer sites). Measurements are planned by ACCO in September 2014 (fab out of test structures). ACCO could measure systematically for the entire nanoCOPS project duration all PCM structures of all wafers after manufacturing.

4.3 ON Semi Data

4.3.1 General description and format of ON Semi Data

ON Semiconductor did provide a full wafer map of measured data of multiple parameters on a FET device, on multiple (2) wafers.The data obtained from ON Semi are organized similarly as data supplied by our project partner ACCO. However, in this situation, the numbers of measurements, which implies also the amount of groups, is much bigger and is 80 for every from seven wafers. The physical quantities under consideration are described rather roughly. Therefore, we decided to choose only two electrical variables, which are encrypted as follows : BV@7A [V] and TaV@7A [S]. Also in this case, the commercial software Statistica, version 10.0 MR1, has been used for the statistical analysis [17].

4.3.2 Results of the statistical analysis

For the data under consideration, BV@7A [V] and TaV@7A [S], the similar statistical analysis has been performed as in the case of the ACCO data. Hence, firstly the assumption on the normal distribution of data is verified using the Shapiro-Wilk test. Based on the result of this test, the homogeneity variance test is applied, if the every group pupulation can be described by the normal distribution. In case of both positive answers, the ANOVA test can be used. Otherwise, the non-parametric Kruskal-Wallis test is applied in order to find the answers for the question formulated in : (i)-(iii).

- WaferMapData__Dev1/ **BV@7A**

Table 1. Descriptive statistics: for all data

Without grouping descriptive statistics (base total)							
	Numbers	**Mean**	**Min.**	**Max.**	**F. quart. Q_1**	**Third quart. Q_3**	**Stand. Dev.**
BV@7A	510	830.3606	798.5000	864.2000	823.6000	835.8000	10.55273

Table 2.Without grouping descriptive statistics, cardinality table: BV@7A Shapiro-Wilk: W=,97540, p =0,0000

Without grouping descriptive statistics (base total)						
	Number	**Cumulative numb.**	**Percentage - valid**	**Cumul. % - valid**	**% of the total - Cases**	**Cumulative %**
790,0000<x<=800,0000	1	1	0.19608	0.1961	0.17857	0.1786
800,0000<x<=810,0000	9	10	1.76471	1.9608	1.60714	1.7857
810,0000<x<=820,0000	53	63	10.39216	12.3529	9.46429	11.2500
820,0000<x<=830,0000	223	286	43.72549	56.0784	39.82143	51.0714
830,0000<x<=840,0000	120	406	23.52941	79.6078	21.42857	72.5000
840,0000<x<=850,0000	85	491	16.66667	96.2745	15.17857	87.6786
850,0000<x<=860,0000	15	506	2.94118	99.2157	2.67857	90.3571
860,0000<x<=870,0000	4	510	0.78431	100.0000	0.71429	91.0714
Lack	50	560	9.80392		8.92857	100.0000

Table 3. Descriptive statistics: for group = 4

With grouping descriptive statistics (base total) : group = 4							
	Numbers	Mean	Min.	Max.	F. quart. Q_1	Third quart. Q_3	Stand. Dev.
BV@7A	6	825.8500	821.4000	841.1000	821.6000	826.2000	7.679258

Table 4. With grouping descriptive statistics: group = 4, cardinality table: BV@7A Shapiro-Wilk: W=,66911, p=,00287

With grouping descriptive statistics (base total) : group = 4						
	Number	Cumulative numb.	Percentage - valid	Cumul. % - valid	% of the total - Cases	Cumulative %
815,0000<x<=820,0000	0	0	0.00000	0.0000	0.00000	0.0000
820,0000<x<=825,0000	4	4	66.66667	66.6667	57.14286	57.1429
825,0000<x<=830,0000	1	5	16.66667	83.3333	14.28571	71.4286
830,0000<x<=835,0000	0	5	0.00000	83.3333	0.00000	71.4286
835,0000<x<=840,0000	0	5	0.00000	83.3333	0.00000	71.4286
840,0000<x<=845,0000	1	6	16.66667	100.0000	14.28571	85.7143
Lack	1	7	16.66667		14.28571	100.0000

In this situation, the Kruskal-Wallis test can be applied due to the fact, that at least one group cannot be described by the normal distribution, for example, group 4. It is not possible to show the result of the Kruskal-Wallis test analysis in this case, since eighty groups are taken into account (eighty measurement points). Therefore, only the conclusion is made: the null hypothesis is rejected, because p = 0.0000 $< \alpha$ = 0.005. The median distribution, together with quartiles Q_1 and Q_3, is shown in Fig. 9.

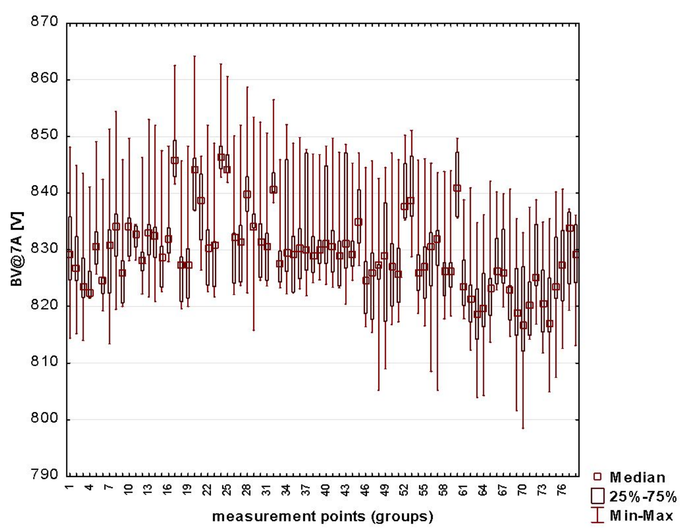


Figure 9: The median distribution together with quartiles Q_1 and Q_3.

Conclusions : (i) the analyzed material considering BV@7A is not homogeneous and it might be anisotropic because the means calculated for particular groups are statistically significant (the green color), (ii) the analyzed data for every measurement point (eighty groups) are spatially independent, (iii) as a result, the measurement data does not belong to the same population.

- WaferMapData__Dev1/ **TaV@7A**

Table 5. Descriptive statistics: for all data

Without grouping descriptive statistics (base total)							
	Numbers	**Mean**	**Min.**	**Max.**	**F. quart. Q_1**	**Third quart. Q_3**	**Stand. Dev.**
TaV@7A	510	77.07843	74.00000	103.0000	76.00000	78.00000	2.042218

Table 6. Without grouping descriptive statistics, cardinality table: TaV@7A Shapiro-Wilk: W=,46347, p =0,0000

Without grouping descriptive statistics (base total)						
	Number	**Cumulative numb.**	**Percentage - valid**	**Cumul. % - valid**	**% of the total - Cases**	**Cumulative %**
70,00000<x<=75,00000	57	57	11.17647	11.1765	11.17647	11.1765
75,00000<x<=80,00000	449	506	88.03922	99.2157	88.03922	99.2157
80,00000<x<=85,00000	0	506	0.00000	99.2157	0.00000	99.2157
85,00000<x<=90,00000	0	506	0.00000	99.2157	0.00000	99.2157
90,00000<x<=95,00000	2	508	0.39216	99.6078	0.39216	99.6078
95,00000<x<=100,0000	1	509	0.19608	99.8039	0.19608	99.8039
100,0000<x<=105,0000	1	510	0.19608	100.0000	0.19608	100.0000
Lack	0	510	0.00000		0.00000	100.0000

Table 7. Descriptive statistics: for group = 29

With grouping descriptive statistics (base total) : group = 29							
	Numbers	**Mean**	**Min.**	**Max.**	**F. quart. Q_1**	**Third quart. Q_3**	**Stand. Dev.**
TaV@7A	5	80.40000	76.00000	93.00000	77.00000	79.00000	7.127412

Table 8. With grouping descriptive statistics: group = 4, cardinality table: TaV@7A Shapiro-Wilk: W=,68310, p=,00632

With grouping descriptive statistics (base total) : group = 29						
	Number	**Cumulative numb.**	**Percentage - valid**	**Cumul. % - valid**	**% of the total - Cases**	**Cumulative %**
70,00000<x<=75,00000	0	0	0.00000	0.0000	0.00000	0.0000
75,00000<x<=80,00000	4	4	80.00000	80.0000	80.00000	80.0000
80,00000<x<=85,00000	0	4	0.00000	80.0000	0.00000	80.0000
85,00000<x<=90,00000	0	4	0.00000	80.0000	0.00000	80.0000
90,00000<x<=95,00000	1	5	20.00000	100.0000	20.00000	100.0000
Lack	0	5	0.00000		0.00000	100.0000

In this case, the Kruskal-Wallis test can be directly used because the group no 29 does not belong to the normal distribution. As in the previous case we do not show the result of the Kruskal-Wallis test analysis for eighty groups. Therefore, only the conclusion is given: the null hypothesis is rejected, due to the fact, that p = 0.00632 $< \alpha$ = 0.005.

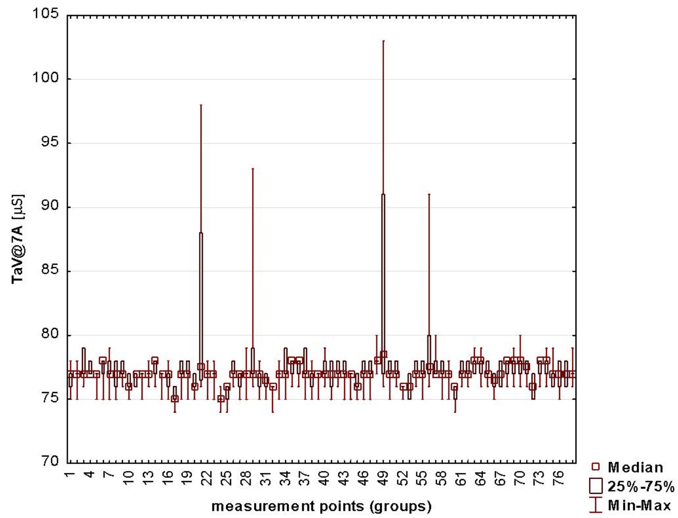


Figure 10: The median distribution together with quartiles Q_1 and Q_3.

Conclusions : (i) the analyzed material taking into account TaV@7A [μS] is not homogeneous and and it might be anisotropic because the means calculated for particular groups are statistically significant (the green color), (ii) the analyzed data for every measurement point (eighty groups) are spatially independent, (iii) as a result, the measurement data does not belong to the same population. The median distribution, together with quartiles Q_1 and Q_3, is shown in Fig. 10.

Acknowledge : We would like to thank our colleagues from the West Pomeranian University of Technology in Szczecin, Poland, for the possibility to perform the statistical data analysis using Statistica software[1].

5 Conclusions

In general, the conclusions, which come directly from the data analysis are as follows:

- To our opinion, the delivered data could be better described, especially for our purpose, including also the needed information about the quality of measurements process, materials under considerations, etc.
- Specifically, in the case of data analysis of the NXP data, without knowledge about the precision/quality of conducted measurement, it is very hard to decide on the null hypothesis for the bivariate normal distribution, based only on the test statistics, because we do not know anything about the measurement errors. We will discuss this more closely with NXP and with BUT for Task T3.3 (Measurements).
- Similarly, for the analyzed ACCO data, the typical α = 0.05 value was used for the verification purpose, but we have assumed that the measurement error should

[1] `http://www.statsoft.com/Products/STATISTICA/Product-Index`.

be lower in the averaged sense. However, for the worst scenario (measurement precision), perhaps also a lower α value should be taken in account.
In section 4.2.3 we already mentioned that ACCO will next measure all sites of wafers (not just 5 wafer sites). Measurements are planned by ACCO in September 2014 (fab out of test structures). ACCO could measure systematically for the entire nanoCOPS project duration all PCM structures of all wafers after manufacturing.

- As far as the ONSEMI data are concerned, again the information on the measurement process and quantities under consideration should be included.

- Both the ACCO data and the ONN data did not have the spatial structure, which we tried to analyse. It is questionable if detailed spatial data with the random structures, which we had in mind, is available at some of our industrial partners. Firstly, the industrial partners seem to have just a few datasets available due to confidentiality reasons. Secondly, the industrial production could still be highly accurate such that no random variations are visible

- We are planning to use the more advanced statistical techniques such as the so-called systematic graphical methods, in particular QQ-plots / normal probability plots and kernel density estimators in order to indicate why something is not distributed according to a certain distribution but for this purpose the quality of delivered data should be higher.

- For the further work in WP2, the modelling of uncertainties should be continued using traditional random distributions (Gaussian, uniform, beta, Gaussian random fields, etc.). We always can get statistical data just for a small subset of problems from our industrial partners. However, since the focus is on our two benchmarks (RF-circuits and Power-MOS) in the nanoCOPS project, it will be more difficult to get data for these examples. Of course, the research on Task T2.6 will continue as planned officially, i.e., we will try to achieve reasonable results.

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