Modified Block-Diagonal Structured Model Order Reduction for Electro-Thermal Problems in Industrial Electronics Simulations

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Summary. Recently, the block-diagonal structured model order reduction method for electro-thermal (ET) coupled problems with many inputs was proposed. After splitting, this MOR method reduces both the electrical and thermal parts separately, using the elimination and block-diagonal structured MOR (BDSM) methods, respectively. However, the reduced electrical part has very dense matrices which is still a computational burden. We propose a modified BDSM-ET method which leads to sparser reduced-order models.

1 Introduction

Spatial discretization of ET coupled problems leads to a nonlinear quadratic dynamical system of the following form,

$$\mathbf{E}\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{x}^T\mathbf{F}\mathbf{x} + \mathbf{B}\mathbf{u}, \, \mathbf{x}(0) = \mathbf{x}_0, \tag{1a}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}, \tag{1b}$$

where $\mathbf{E} \in \mathbb{R}^{n\times n}$ is singular, indicating that the system in (1) is a system of differential-algebraic equations (DAEs), and $\mathbf{A} \in \mathbb{R}^{n\times n}, \mathbf{B} \in \mathbb{R}^{n\times m}, \mathbf{C} \in \mathbb{R}^{\ell\times n}, \mathbf{D} \in \mathbb{R}^{\ell\times m}$, while the tensor $\mathbf{F} = \left[\mathbf{F}_1^T,\dots,\mathbf{F}_n^T\right]^T$ is a 3-D array of n matrices $\mathbf{F}_i \in \mathbb{R}^{n\times n}$. Each element in $\mathbf{x}^T\mathbf{F}\mathbf{x} \in \mathbb{R}^n$ is a scalar $\mathbf{x}^T\mathbf{F}_i\mathbf{x} \in \mathbb{R}, i = 1,\dots,n$. The state vector $\mathbf{x} = (\mathbf{x}_v^T, \mathbf{x}_T^T)^T \in \mathbb{R}^n$ includes the nodal voltages $\mathbf{x}_v \in \mathbb{R}^{n_v}$, and the nodal temperatures $\mathbf{x}_T \in \mathbb{R}^{n_T}$. $\mathbf{u} = \mathbf{u}(t) \in \mathbb{R}^m$ and $\mathbf{y} = \mathbf{y}(t) \in \mathbb{R}^\ell$ are the inputs (excitations) and the desired outputs (observations), respectively. We assume system (1) to be solvable, that is, the matrix pencil $\lambda\mathbf{E} - \mathbf{A}$ is regular, $\forall\, \lambda \in \mathbb{C}$. For simplicity, we assume (1) to be weakly coupled, and has the following matrix structures,
$\mathbf{E} = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{E}_T \end{pmatrix}, \mathbf{A} = \begin{pmatrix} \mathbf{A}_v & 0 \\ 0 & \mathbf{A}_T \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \mathbf{B}_v & 0 \\ 0 & \mathbf{B}_T \end{pmatrix},$
$\mathbf{C} = \begin{pmatrix} \mathbf{C}_v & \mathbf{C}_T \end{pmatrix}, \mathbf{D} = \begin{pmatrix} \mathbf{D}_v & \mathbf{D}_T \end{pmatrix}, \mathbf{u} = \begin{pmatrix} \mathbf{u}_v^T & \mathbf{u}_T^T \end{pmatrix}^T$, with $\mathbf{A}_v \in \mathbb{R}^{n_v\times n_v}, \mathbf{B}_v \in \mathbb{R}^{n_v\times m/2}, \mathbf{E}_T \in \mathbb{R}^{n_T\times n_T}, \mathbf{A}_T \in \mathbb{R}^{n_T\times n_T}, \mathbf{B}_T \in \mathbb{R}^{n_T\times m/2}, \mathbf{C}_v \in \mathbb{R}^{\ell\times n_v}, \mathbf{C}_T \in \mathbb{R}^{\ell\times n_T}, \mathbf{D}_v \in \mathbb{R}^{\ell\times m/2}, \mathbf{D}_T \in \mathbb{R}^{\ell\times m/2}$, and $\mathbf{u}_v, \mathbf{u}_T \in \mathbb{R}^{m/2}$. Then the system (1) can be written as

$$\mathbf{A}_v\mathbf{x}_v = -\mathbf{B}_v\mathbf{u}_v, \tag{2a}$$

$$\mathbf{E}_T\mathbf{x}_T' = \mathbf{A}_T\mathbf{x}_T + \mathbf{x}_v^T\mathbf{F}_T\mathbf{x}_v + \mathbf{B}_T\mathbf{u}_T, \tag{2b}$$

$$\mathbf{y} = \mathbf{C}_v\mathbf{x}_v + \mathbf{C}_T\mathbf{x}_T + \mathbf{D}_v\mathbf{u}_v + \mathbf{D}_T\mathbf{u}_T, \tag{2c}$$

with initial condition $\mathbf{x}_T(0) = \mathbf{x}_{T_0}$ and $\mathbf{F}_T \in \mathbb{R}^{n_v\times n_v\times n_T}$ being a tensor. In this work, we consider MOR of system in (2) with large ℓ and m. It is well known that, models with numerous inputs and outputs are challenging for MOR, and most MOR methods produce large, dense reduced-order models (ROMs) for such systems.

In [1], the BDSM-ET method was proposed to overcome this problem, leading to a ROM

$$\mathbf{A}_{v_r}\mathbf{x}_{v_r} = -\mathbf{B}_{v_r}\mathbf{u}_v, \tag{3a}$$

$$\mathbf{E}_{T_r}\mathbf{x}_{T_r}' = \mathbf{A}_{T_r}\mathbf{x}_{T_r} + \mathbf{x}_{v_r}^T\mathbf{F}_{T_r}\mathbf{x}_{v_r} + \mathbf{B}_{T_r}\mathbf{u}_T, \tag{3b}$$

$$\mathbf{y}_r = \mathbf{C}_{v_r}\mathbf{x}_{v_r} + \mathbf{C}_{T_r}\mathbf{x}_{T_r} + \mathbf{D}_{v_r}\mathbf{u}_v + \mathbf{D}_{T_r}\mathbf{u}_T, \tag{3c}$$

where $\mathbf{A}_{v_r} \in \mathbb{R}^{r_v\times r_v}$, $\mathbf{B}_{v_r} \in \mathbb{R}^{r_v\times m/2}$, $\mathbf{E}_{T_r} \in \mathbb{R}^{r_T\times r_T}$, $\mathbf{A}_{T_r} \in \mathbb{R}^{r_T\times r_T}$, $\mathbf{B}_{T_r} \in \mathbb{R}^{r_T\times m/2}$, $\mathbf{C}_{v_r} \in \mathbb{R}^{\ell\times r_v}$, $\mathbf{C}_{T_r} \in \mathbb{R}^{\ell\times r_T}$, $\mathbf{D}_{v_r} = \mathbf{D}_v$, $\mathbf{D}_{T_r} = \mathbf{D}_T$, $\mathbf{F}_{T_r} \in \mathbb{R}^{r_v\times r_v\times r_T}$, such that the reduced order, $r = r_v + r_T \ll n$, and the approximation error $\|\mathbf{y} - \mathbf{y}_r\|$ is small with respect to a suitable norm $\|.\|$. However, matrix $\mathbf{A}_{v_r}$ and tensor $\mathbf{F}_{T_r}$ are dense which is still a computational burden. In the next section, we propose a modified BDSM-ET method which leads to sparser ROMs.

2 Proposed modified BDSM-ET method

We propose to first apply the superposition principle to both the electrical (2a) and thermal (2b) subsystems, respectively. Then we conduct MOR and generate a block-diagonal structured sparse ROM. Without loss of generality, assume that the input matrices $\mathbf{B}_v$ and $\mathbf{B}_T$ have no zero columns so that, they can be split into $\mathbf{B}_v = \sum_{i=1}^{m/2}\mathbf{B}_{v_i}, \mathbf{B}_T = \sum_{i=1}^{m/2}\mathbf{B}_{T_i}$ where $\mathbf{B}_{v_i} \in \mathbb{R}^{n_v\times m/2}, \mathbf{B}_{T_i} \in \mathbb{R}^{n_T\times m/2}$ are column rank-1 matrices defined as $\mathbf{B}_{k_i}(:,j) = \begin{cases} \mathbf{b}_{k_i} \in \mathbb{R}^{n_k}, & \text{if } j = i, \\ 0, & \text{otherwise,} \end{cases}$,

$i=1,\dots,m/2$ and $k=v,T$. Using the above input matrix splitting, the superposition principle can be applied to both the electrical and thermal subsystems of system (2), separately as follows.

Reduction of the electrical subsystem

Using the superposition principle, the electrical subsystem in (2) can be split into $m/2$ subsystems

$$\mathbf{A}_v \mathbf{x}_{v_i} = -\mathbf{B}_{v_i}\mathbf{u}_v, \quad \mathbf{y}_{v_i} = \mathbf{C}_v \mathbf{x}_{v_i}, \tag{4}$$

$i=1,\dots,m/2$, where $\mathbf{y}_v = \sum_{i=1}^{m/2}\mathbf{y}_{v_i}$. Let blkdiag denote the block-diagonal matrix defined by the input arguments. The next step is to reduce the dimension of each subsystem in (4). This is done by using reordering and elimination techniques for each subsytem. Reordering the entries of each subsystem in (4) such that the first $n_{v_e}^{(i)}$ rows correspond to the nonzero rows of the input matrix $\mathbf{B}_{v_i}$ and the rest $n_{v_I}^{(i)}$ rows correspond to the internal nodes, leads to a partitioned subsystem

$$\begin{aligned} \begin{pmatrix} \mathbf{A}_{v_{11}}^{(i)} & \mathbf{A}_{v_{12}}^{(i)} \\ \mathbf{A}_{v_{12}}^{(i)^T} & \mathbf{A}_{v_{22}}^{(i)} \end{pmatrix} \begin{pmatrix} \mathbf{x}_{v_e}^{(i)} \\ \mathbf{x}_{v_I}^{(i)} \end{pmatrix} &= -\begin{pmatrix} \mathbf{B}_{v_e}^{(i)} \\ 0 \end{pmatrix}\mathbf{u}_v, \\ \mathbf{y}_{v_i} &= \begin{pmatrix} \mathbf{C}_{v_e}^{(i)} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x}_{v_e}^{(i)} \\ \mathbf{x}_{v_I}^{(i)} \end{pmatrix}, \end{aligned} \tag{5}$$

where $\mathbf{x}_{v_e}^{(i)} \in \mathbb{R}^{n_{v_e}^{(i)}}$ and $\mathbf{x}_{v_I}^{(i)} \in \mathbb{R}^{n_{v_I}^{(i)}}$ represent the port and the internal nodal voltages, respectively, and $n_v = n_{v_e}^{(i)} + n_{v_I}^{(i)}$, $i=1,\dots,m/2$. Eliminating all internal nodes from (5) leads to the ROM of (4) as below

$$\mathbf{A}_{v_{r_i}}\mathbf{x}_{v_{r_i}} = \mathbf{B}_{v_{r_i}}\mathbf{u}_v, \quad \mathbf{y}_{v_{r_i}} = \mathbf{C}_{v_{r_i}}\mathbf{x}_{v_{r_i}}, \tag{6}$$

where $\mathbf{x}_{v_{r_i}} = \mathbf{x}_{v_e}^{(i)} \in \mathbb{R}^{r_{v_i}}$, $\mathbf{B}_{v_{r_i}} = -\mathbf{B}_{v_e}^{(i)} \in \mathbb{R}^{r_{v_i}\times m/2}$, $\mathbf{A}_{v_{r_i}} = \mathbf{A}_{v_{11}}^{(i)} - \mathbf{A}_{v_{12}}^{(i)}\mathbf{W}_{v_i} \in \mathbb{R}^{r_{v_i}\times r_{v_i}}$, $\mathbf{C}_{v_{r_i}} = \mathbf{C}_{v_e}^{(i)} \in \mathbb{R}^{\ell\times r_{v_i}}$, $\mathbf{W}_{v_i} = \mathbf{A}_{v_{22}}^{(i)^{-1}}\mathbf{A}_{v_{12}}^{(i)^T} \in \mathbb{R}^{n_{v_I}^{(i)}\times n_{v_e}^{(i)}}$, and $r_{v_i} = n_{v_e}^{(i)} \ll n_v$. Hence, the reduced electrical subsystem can be reformulated as the parallel connection of the reduced-order subsystems in (6). Consequently, it can be equivalently transformed into a block-diagonal reduced system of dimension $r_v = \sum_{i=1}^{m/2} r_{v_i}$. Thus, the reduced-order electrical subsystem in the form of (3a), has the following matrices, $\mathbf{A}_{v_r} = \text{blkdiag}(\mathbf{A}_{v_{r_1}},\dots,\mathbf{A}_{v_{r_{m/2}}})$, $\mathbf{C}_{v_r} = (\mathbf{C}_{v_{r_1}},\dots,\mathbf{C}_{v_{r_{m/2}}})$, $\mathbf{B}_{v_r} = (\mathbf{B}_{v_{r_1}}^T,\dots,\mathbf{B}_{v_{r_{m/2}}}^T)^T$.

Reduction of the thermal subsystem

We observe that, splitting (2a) into $m/2$ subsystems in (4) induces the splitting of the nonlinear part in the thermal part (2b). When the approximation $\left(\sum_{i=1}^{m/2}\mathbf{x}_{v_i}^T\right)\mathbf{F}_T\left(\sum_{i=1}^{m/2}\mathbf{x}_{v_i}\right) \approx \sum_{i=1}^{m/2}\mathbf{x}_{v_i}^T\mathbf{F}_T\mathbf{x}_{v_i}$ is introduced, the thermal subsystem (2b) can be written as

$$\mathbf{E}_T\mathbf{x}_T' = \mathbf{A}_T\mathbf{x}_T + \xi_v^T\mathscr{F}_T\xi_v, + \mathbf{B}_T\mathbf{u}_T, \tag{7}$$

where we have used the equality $\sum_{i=1}^{m/2}\mathbf{x}_{v_i}^T\mathbf{F}_T\mathbf{x}_{v_i} = \xi_v^T\mathscr{F}_T\xi_v$, where $\mathscr{F}_T = \{\mathscr{F}_{T_1},\dots,\mathscr{F}_{T_{n_T}}\} \in \mathbb{R}^{\tilde{n}_v\times\tilde{n}_v\times n_T}$, $\tilde{n}_v = mn_v/2$, being a 3D-array of n_T block-diagonal matrices $\mathscr{F}_{T_i} = \text{blkdiag}(\mathbf{F}_{T_i},\dots,\mathbf{F}_{T_i}) \in \mathbb{R}^{\tilde{n}_v\times\tilde{n}_v}$, $\mathbf{F}_{T_i} \in \mathbb{R}^{n_v\times n_v}$ and $\xi_v = (\mathbf{x}_{v_1}^T,\dots,\mathbf{x}_{v_{m/2}}^T)^T$. We assume such an approximation to be possible. Though this seems like a strong assumption, we have observed it to be valid in some of our applications.

Also the reduction of the algebraic part induces a reduction in the differential part leading to

$$\mathbf{E}_T\mathbf{x}_T' = \mathbf{A}_T\mathbf{x}_T + \xi_{v_r}^T\mathbf{F}_{T_r}\xi_{v_r} + \mathbf{B}_T\mathbf{u}_T, \tag{8}$$

with $\mathbf{F}_{T_r} = \{\mathscr{F}_{T_{r_1}},\dots,\mathscr{F}_{T_{r_{n_T}}}\} \in \mathbb{R}^{r_v\times r_v\times n_T}$ being a 3D-array of n_T reduced order block-diagonal matrices $\mathscr{F}_{T_{r_i}} = \text{blkdiag}(\mathbf{F}_{T_{r_i}},\dots,\mathbf{F}_{T_{r_i}}) \in \mathbb{R}^{r_v\times r_v}$, where $\mathbf{F}_{T_{r_i}} = \mathbf{F}_{T_{11}}^{(i)} - \mathbf{W}_{v_i}^T\mathbf{F}_{T_{21}}^{(i)} - \mathbf{F}_{T_{12}}^{(i)}\mathbf{W}_{v_i} + \mathbf{W}_{v_i}^T\mathbf{F}_{T_{22}}^{(i)}\mathbf{W}_{v_i} \in \mathbb{R}^{r_{v_i}\times r_{v_i}}$. Since system (8) can also be split into $m/2$ subsystems, the thermal state $\mathbf{x}_T$ of system (8) can be reduced using the BDSM-ET method proposed in [1]. Hence, the reduced thermal system in (3b) also has block-diagonal structured matrices given by, $\mathbf{E}_{T_r} = \mathbf{V}^{\mathrm{T}}\mathscr{E}_T\mathbf{V}$, $\mathbf{A}_{T_r} = \mathbf{V}^{\mathrm{T}}\mathscr{A}_T\mathbf{V}$, $\mathbf{B}_{T_r} = \mathbf{V}^{\mathrm{T}}\mathscr{B}_T$, $\mathbf{C}_{T_r} = \mathscr{C}_T\mathbf{V}$, where $\mathscr{E}_T = \text{blkdiag}(\mathbf{E}_T,\dots,\mathbf{E}_T)$, $\mathscr{C}_T = (\mathbf{C}_T,\dots,\mathbf{C}_T)$ $\mathscr{A}_T = \text{blkdiag}(\mathbf{A}_T,\dots,\mathbf{A}_T)$, $\mathscr{B}_T = (\mathbf{B}_{T_1}{}^T,\dots,\mathbf{B}_{T_{m/2}}{}^T)^T$, $\mathbf{V} = \text{blkdiag}(\mathbf{V}^{(1)},\dots,\mathbf{V}^{(m/2)})$. The projection matrices $\mathbf{V}^{(i)}$ can be constructed as in [2],

$$\text{range}(\mathbf{V}^{(i)}) = \text{span}\{\mathbf{R}_i, \mathbf{M}\mathbf{R}_i, \cdots, \mathbf{M}^{r_{T_i}-1}\mathbf{R}_i\},\ r_{T_i} \ll n_T,$$

where $\mathbf{M} = (s_0\mathbf{E}_T - \mathbf{A}_T)^{-1}\mathbf{E}_T \in \mathbb{R}^{n_T\times n_T}$, and $\mathbf{R}_i = (s_0\mathbf{E}_T - \mathbf{A}_T)^{-1}\mathbf{b}_{T_i} \in \mathbb{R}^{n_T}$, $i=1,\dots m/2$. Here $s_0 \in \mathbb{C}$ is chosen arbitrarily. Hence, the order of the reduced thermal subsystem (3b) is $r_T = \sum_{i=1}^{m/2} r_{T_i}$.

3 Conclusion

By construction, the modified BDSM-ET method leads to sparser ROMs than the BDSM-ET method proposed in [1] with accurate ROMs. We have compared the two methods using ET coupled problems with many inputs from industry.

References

1. N. Banagaaya, L. Feng, W. Schoenmaker, P. Meuris, A. Wieers, R. Gillon, and P. Benner. Model order reduction for nanoelectronics coupled problems with many inputs. In *Proceedings 2016 Design, Automation & Test in Europe Conference & Exhibition (DATE)*, pages 313–318, 2016.
2. U. Baur, P. Benner, and L. Feng. Model order reduction for linear and nonlinear systems: A system-theoretic perspective. *Arch. Comput. Methods Eng.*, 21(4):331–358, 2014.