Iterative Schemes for Coupled Multiphysical Problems in Electrical Engineering

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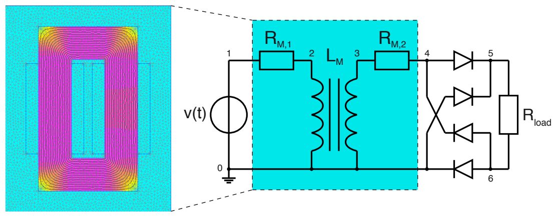
Summary. This contributions addresses iterative coupling schemes for coupled model descriptions in computational electromagnetics. Theoretical issues of accuracy, stability and numerical efficiency of the resulting formulations are addressed along with advantages and disadvantages of the various approaches. Three application examples are given: field-circuit coupling, a mechanical-electromagnetic and thermal-electromagnetic problem.



(a) 2D transformer model (b) rectifier circuit with embeded PDE model

Fig. 1. Simple field-circuit coupled problem.

1 Introduction

Today, due to increased accuracy of modeling and simulation, multiphysical problems become more and more important in many engineering applications. Often a monolithic approach, i.e., the solution of all subproblems at once, is cumbersome or even impossible because incompatible algorithms or software packages are involved. Thus simulation engineers need to couple subproblems in an efficient and stable way, where subdomains are solved separately. This introduces a splitting error, which is mitigated by an iterative procedure.

In this contribution we like to advertise the increased accuracy and stability due to iterative procedures by discussing three examples: field-circuit coupling in Section 1, a mechanical-electromagnetic problem in Section 2 and finally a thermal-electromagnetic problem in Section 3. In the full contribution also implementation issues and the practical relevance of those iteration schemes will be discussed.

2 Field-Circuit Problem

For field-circuit coupled models of electrical energy transducers, two general approaches are well established. A first approach consists of extracting lumped parameters or surrogate models from a field model and inserting these as a netlist into a Spice-like circuit simulator. This is circumvented by monolithic coupling, where field and circuit models are solved together. We propose a particular synthesis: the parameter extraction is applied iteratively on time intervals. The eddy-current field problem on Ω is

$$\sigma\partial_t \mathbf{a}^{(n)} + \nabla\times\left(\nu(|\nabla\times\mathbf{a}^{(n)}|)\,\nabla\times\mathbf{a}^{(n)}\right) = \chi\mathbf{j}^{(n)}$$

where $\mathbf{a}^{(n)}$ is the magnetic vector potential after the n-th iteration (with homogeneous Dirichlet conditions), σ and ν are conductivity and reluctivity, respectively and the winding functions $\chi = [\chi_1,\ldots,\chi_k,\ldots,\chi_K]^\top$ are functions of space that distribute the lumped currents $\mathbf{j}$ in the 3D domain. The circuit coupling is established via integration

$$\partial_t \int_\Omega \chi_k \mathbf{a}^{(n)}\,\mathrm{d}x + R_k j_k^{(n)} = v_k^{(n-1)} \qquad k=1,\ldots,K$$

to the circuit system of differential algebraic equations

$$\begin{aligned}
\mathbf{A}_\mathrm{C}\partial_t\mathbf{q}_\mathrm{C}(\mathbf{A}_\mathrm{C}^\mathrm{T}\mathbf{u}^{(n)},t) + \mathbf{A}_\mathrm{R}\mathbf{g}_\mathrm{R}(\mathbf{A}_\mathrm{R}^\mathrm{T}\mathbf{u},t) + \mathbf{A}_\mathrm{L}\mathbf{i}_\mathrm{L}^{(n)}&\\
+\mathbf{A}_\mathrm{M}\mathbf{j}^{(n)} + \mathbf{A}_\mathrm{V}\mathbf{i}_\mathrm{V}^{(n)} + \mathbf{A}_\mathrm{I}\mathbf{i}_\mathrm{s}(t) &= 0,\\
\partial_t\Phi_\mathrm{L}(\mathbf{i}_\mathrm{L}^{(n)},t) - \mathbf{A}_\mathrm{L}^\mathrm{T}\mathbf{u} &= 0,\\
\mathbf{A}_\mathrm{V}^\mathrm{T}\mathbf{u} - \mathbf{v}_\mathrm{s}(t) &= 0,
\end{aligned}$$

with incidence matrices $\mathbf{A}_*$ where $\mathbf{v}_* = \mathbf{A}_*^\mathrm{T}\mathbf{u}$ and constitutive laws for conductances, inductances and capacitances (functions with subscripts R, L and C), independent sources $\mathbf{i}_\mathrm{s}$ and $\mathbf{v}_\mathrm{s}$, unknowns are the potentials $\mathbf{u}$ and currents $\mathbf{i}_\mathrm{L}$ and $\mathbf{i}_\mathrm{V}$.

In the full paper the convergence, [1,2], of this iteration scheme and tailored time integration will be discussed. It will be shown that the optimal time integration order depends on the iteration counter n, [6].

3 Field-Mechanical Problem

The Lorentz detuning of an accelerating cavity, which is the change of the resonant frequency due to the mechanical deformation of the cavity wall induced by the

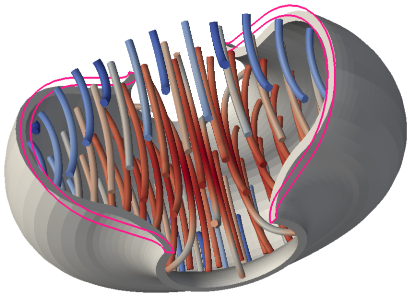


Fig. 2. One cavity cell with field lines and exaggerated deformation, [3]

electromagnetic pressure is a coupled electromagnetic-mechanical problem. In a first step, Maxwell's eigenproblem is solved

$$\nabla \times \left(\frac{1}{\mu_0}\nabla \times \mathbf{e}^{(n)}\right) = \omega_0^2 \varepsilon_0 \mathbf{e}^{(n)} \quad \text{on } \Omega^{(n-1)}$$

where $\mathbf{e}$ is the phasor of the electric field with adequate boundary conditions; μ_0 and ε_0 are the permeability and permittivity of vacuum. From this the magnetic field $\mathbf{h}$ can be obtained. Both fields create a radiation pressure at the boundary of $\Omega^{(n-1)}$

$$p^{(n)} = -\frac{1}{2}\varepsilon_0 e_{\perp}^{(n)} \left(e_{\perp}^{(n)}\right)^* + \frac{1}{2}\mu_0 \mathbf{h}_{\parallel}^{(n)} \left(\mathbf{h}_{\parallel}^{(n)}\right)^*$$

which gives raise to the linear elasticity problem in the wall of the cavity

$$\nabla \cdot \left(2\eta \nabla^{(S)}\mathbf{u} + \lambda \mathbf{I} \nabla \cdot \mathbf{u}\right) = 0$$

for the displacement $\mathbf{u}^{(n)}$ where $p^{(n)}$ is a boundary condition on the inner boundary. We denote by $\nabla^{(S)}$ the symmetric gradient, while η and λ are the Lamé constants. Finally a deformed domain

$$\Omega^{(n)} \equiv \left\{\mathbf{x} + \mathbf{u}^{(n)}(\mathbf{x}), \mathbf{x} \in \Omega^{(0)}\right\},$$

is derived from the initial domain $\Omega^{(0)}$ and the iteration can be restarted with the computation of an eigenvalue. In the full paper this scheme will be discussed in more details. The focus will be on the spatial discretization with Isogeometric Analyses using Non-Uniform Rational B-Spline (NURBS) and De-Rham-conforming B-Splines [3].

4 Field-Thermal Problem

In the previous sections we have discussed the mutual coupling of transient and frequency-domain to static problems. The third example revisits the well-known iterative coupling of frequency to time domain problems. Again, the electromagnetic field is given by the curl-curl equation, however since we are in frequency domain we can regard displacement currents

$$\varepsilon\omega^2 \mathbf{a}^{(n)} + \sigma(T^{(n-1)}) j\omega \mathbf{a}^{(n)} + \nabla \times (\nu \nabla \times \mathbf{a}^{(n)}) = \chi \mathbf{j}.$$

where $\mathbf{a}$ is now a complex phasor. This is coupled to the heat equation

$$\rho\, c\, \partial_t T^{(n)} = \nabla \cdot (k \nabla T^{(n)}) + Q(\mathbf{a}^{(n)}, t)$$

by the Joule losses Q, where k is the heat conductivity, ρ the density and c the specific heat capacity. Besides the electric conductivity σ, all material parameters are constant. The important modelling step is to relax the coupling of both problems by introducing a averaged heat source

$$\bar{Q}^{(n)} := \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} Q(\mathbf{a}^{(n)}(t), t)\mathrm{d}s.$$

obtained by converting the vector potential a back to the time domain. Convergence will be discussed in view of the works [4, 5] and the fractional step method, [7].

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